

Lectures in Paleomagnetism, 2005
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Chapter 2

The Geomagnetic Field

Suggested Supplemental Reading

pp 3-7; 10-11: Butler (1992)

To learn more:

Chapters 1 & 2: Merrill et al. (1996)

2.1 The Earth's Magnetic Field

One of the major efforts in paleomagnetism has been to study ancient geomagnetic fields. Because human measurements only extend back a few centuries, paleomagnetism remains the only way to investigate ancient field behavior. Because of the application of paleomagnetism to geomagnetism, it is useful for students of paleomagnetism to understand something about the geomagnetic field. In this lecture we review the general properties of the Earth's present magnetic field.

The geomagnetic field is generated by convection currents in the liquid outer core of the Earth which is composed of iron, nickel and some unknown lighter component(s). The source of energy for this convection is not known for certain, but is thought to be partly cooling of the core and partly the buoyancy of the iron/nickel liquid outer core caused by freezing out of the pure iron inner core. Motions of this conducting fluid are controlled by the buoyancy of the liquid, the spin of the Earth about its axis and by the interaction of the conducting fluid with the magnetic field (in a horrible non-linear fashion). Solution of the equations for the fluid motions and resulting magnetic fields is a challenging computational task, but it is known that these motions act as a self-sustaining dynamo and create an enormous magnetic field.

2.1.1 Reference magnetic field

For many purposes, it is useful to have a compact representation of the spatial distribution of the geomagnetic field for a particular time. It is often handy to have a mathematical approximation for the field along with estimates for rates of change such that field vectors can be accurately estimated at a given place at a given time (within a few hundred years at least). As we learned in Lecture 1, the magnetic field at the Earth's surface can be approximated by a scalar potential field, and this scalar potential field satisfies Laplace's Equation:

$$\nabla^2 \psi_m = 0$$

This can be rewritten as:

$$\frac{1}{r} \frac{\partial^2}{\partial r^2} (r\psi_m) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi_m}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial \psi_m}{\partial \phi^2} = 0$$

One solution to this equation is:

$$\psi_m = \frac{U(r)}{r} P(\theta) Q(\phi).$$

For the geomagnetic field, this is usually written as the scalar potential at radius r , co-latitude θ , longitude ϕ :

$$\begin{aligned} \psi(r, \theta, \phi) = \frac{a}{\mu_o} \sum_{l=1}^{\infty} \sum_{m=0}^l P_l^m(\cos \theta) \left[\left((g_e)_l^m \left(\frac{r}{a} \right)^l + (g_i)_l^m \left(\frac{a}{r} \right)^{l+1} \right) \cos m\phi \right. \\ \left. + \left((h_e)_l^m \left(\frac{r}{a} \right)^l + (h_i)_l^m \left(\frac{a}{r} \right)^{l+1} \right) \sin m\phi \right], \end{aligned} \quad (2.1)$$

where g and h are *gauss coefficients* calculated for a particular year and are given in units of nT, or magnetic flux (note the μ_o in the equation converting from tesla $[B]$ to $\text{Am}^{-1} [H]$). The e and i subscripts indicate fields of external or internal origin and a is the radius of the Earth (6.371×10^6 m), μ_o is the permeability of free space (see Table 1.1 from Lecture 1) and the P_l^m 's are proportional to the Legendre polynomials, normalized according to the convention of Schmidt (see suggested reading for more details). The Schmidt polynomials are increasingly wiggly functions of the argument $\cos \theta$.

We show three examples in Fig. 2.2 of the inclinations of the vector fields with their surface harmonics as insets. These are the axial ($m = 0$) dipole ($l = 1$), quadrupole ($l = 2$) and octupole ($l = 3$) terms whose contributions are determined by g_1^0, g_2^0 and g_3^0 respectively. The associated polynomials are:

$$P_1^0 = \cos \theta, \quad P_2^0 = \frac{1}{4}(3 \cos 2\theta + 1), \quad P_3^0 = \frac{1}{8}(5 \cos 3\theta + 3 \cos \theta).$$

and are shown in Figure 2.1.

If the axial dipole field produced by the harmonic function in Fig. 2.2a were turned on its side with the north pole part pointing to the Greenwich meridian, the contribution would be determined by h_1^0 coefficient, and if it were at 90°E , it would be the h_1^1 coefficient. Therefore, the total dipole contribution would be the vector sum of the axial and two equatorial dipole terms or $\sqrt{g_1^0{}^2 + h_1^0{}^2 + h_1^1{}^2}$. The total quadrupole contribution ($l = 2$) combines five coefficients and the total octupole ($l = 3$) contribution combines seven coefficients.

In general, terms for which the difference between the subscript (l) and the superscript (m) is odd (e.g., the axial dipole g_1^0 and octupole g_3^0) produce magnetic fields that are asymmetric about the equator, while those for which the difference is even (e.g., the axial quadrupole g_2^0) have symmetric fields. In Fig. 2.2a we show the inclinations produced by a purely dipole field of the same sign as the present day field. The inclinations are all positive (down) in the northern hemisphere and negative (up) in the southern hemisphere. In contrast, inclinations produced by a purely quadrupolar field (Fig. 2.2b) are down at the poles and up at the equator. The map of inclinations produced by a purely axial octupolar field (Fig. 2.2c) are again asymmetric about the

2.1. THE EARTH'S MAGNETIC FIELD

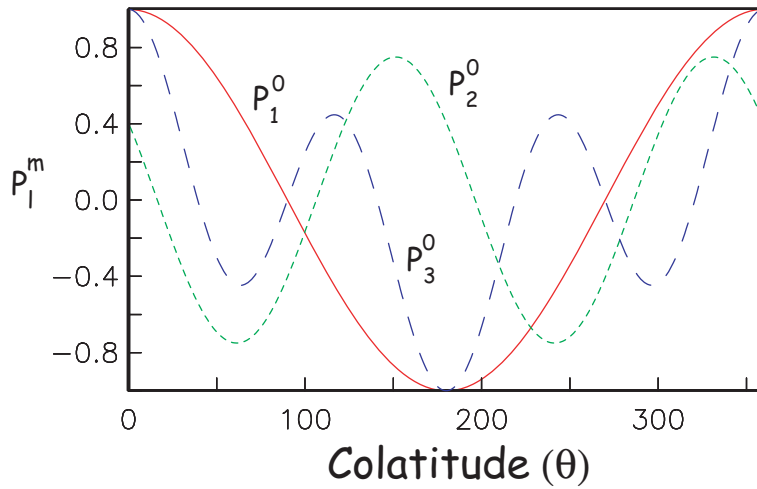


Figure 2.1: Schmidt polynomials

equator with vertical directions of opposite signs at the poles separated by bands with the opposite sign at mid-latitudes.

Getting back to observations of the geomagnetic field itself. It is a vector field, hence at every point, there is a direction and intensity (see Figure 2.3). A vector in three dimensions requires three parameters to define it fully no matter what coordinate system you choose. In cartesian coordinates these would be, for example, x, y, z or x_1, x_2 and x_3 . Depending on the particular problem at hand, some coordinate systems are more suitable to use because they have the symmetry of the problem built into them. We will be using several coordinate systems in addition to the cartesian one and we will need to convert among them at will.

2.1.2 Components of magnetic vectors

The three elements of a magnetic vector that will be used most frequently are magnitude B (or sometimes H or M), declination D and inclination I , as shown in Figure 2.3. The convention used in this set of lectures is that axes are denoted $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3$, while the components along the axes are x_1, x_2, x_3 . In the geographic frame of reference, positive $\mathbf{X}_1$ is to the north, $\mathbf{X}_2$ is east and $\mathbf{X}_3$ is vertically down in keeping with a right-hand rule; components of $\mathbf{B}$, for example, can alternatively be designated B_N, B_E, B_V .

From Figure 2.3 we see how to convert from the angular coordinate system of declination, inclination and total field magnitude to cartesian coordinate systems, using a little trigonometry, *i.e.*,

$$B_H = B \cos I = \sqrt{B_E^2 + B_N^2} \quad \text{and} \quad B_V = B \sin I = x_3. \quad (2.2)$$

The horizontal component can also be projected onto the North ($\mathbf{X}_1$) and East ($\mathbf{X}_2$) axes (the directions in which measurements are often made), *i.e.*,

$$B_N = x_1 = B \cos I \cos D \quad \text{and} \quad B_E = x_2 = B \cos I \sin D. \quad (2.3)$$

Equations 2.2 and 2.3 work equally well for components of magnetization.

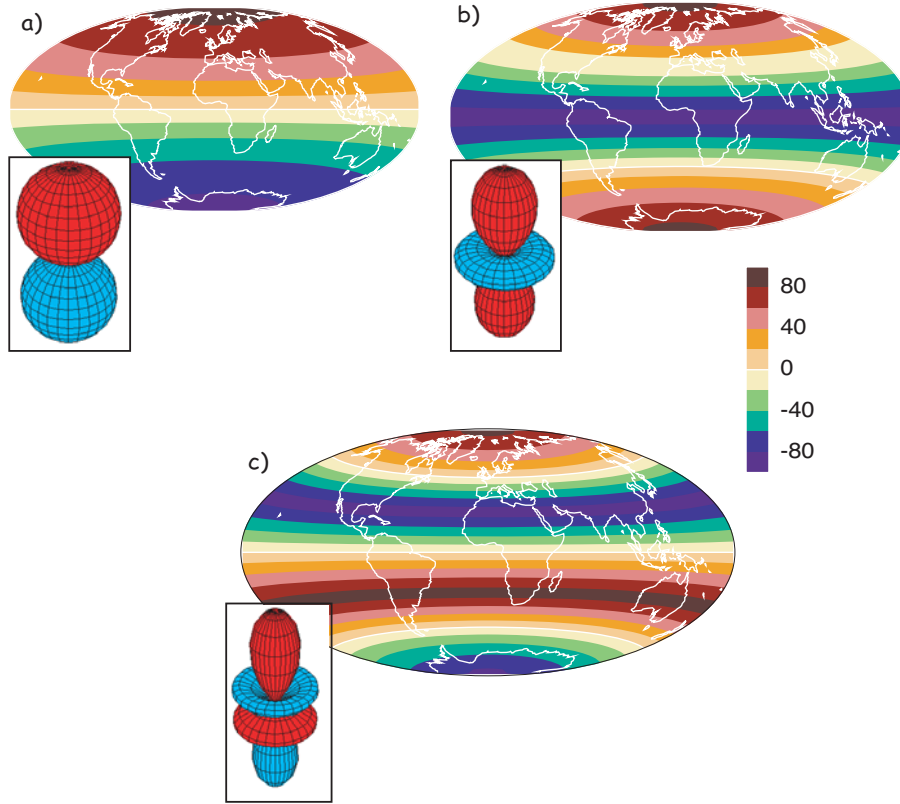


Figure 2.2: Examples of surface harmonics (insets) and maps of the associated patterns for global inclinations. a) Dipole, b) Quadrupole, c) Octupole. [Figure from Tauxe, 2005.]

If you have the cartesian coordinates of $\mathbf{B}$ (or $\mathbf{H}$ or $\mathbf{M}$), they can be transformed to the geomagnetic elements D , I and B :

$$\begin{aligned} B &= \sqrt{x_1^2 + x_2^2 + x_3^2}, \\ D &= \tan^{-1}(x_2/x_1), \\ I &= \sin^{-1}(x_3/B). \end{aligned} \quad (2.4)$$

Be careful of the sign ambiguity of the tangent function. You may end up in the wrong quadrant and have to add 180°

Recalling Lecture 1 (including the appendix), once the scalar potential ψ_m is known, the components of the magnetic field can be calculated from the fact that $B = \nabla\psi_m$, so in spherical coordinates:

$$B_N = -\frac{1}{r} \frac{\partial \psi_m}{\partial \theta}, B_E = -\frac{1}{r \sin \theta} \frac{\partial \psi_m}{\partial \phi}, B_V = -\frac{\partial \psi_m}{\partial r}. \quad (2.5)$$

where r , θ , ϕ are radius, co-latitude (degrees away from the North pole) and longitude, respectively. Here, B_V is positive down B_N is positive to the north, the opposite of H_r and H_θ as defined in Lecture 1. Note that Equation 2.1 is in units of tesla, not Am^{-1} .

It is also true that if the magnetic vector field is known, the potential can be derived. In practice, the gauss coefficients for a particular reference field are estimated by least-squares fitting

2.1. THE EARTH'S MAGNETIC FIELD

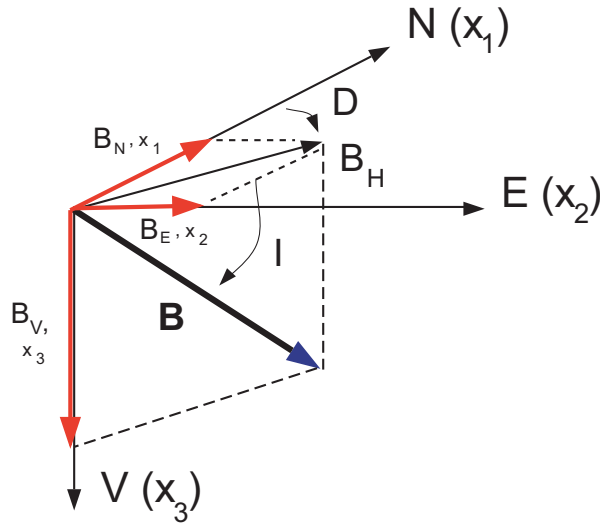


Figure 2.3: Components of the geomagnetic field vector $\mathbf{B}$. B_H is the projection of the field vector $\mathbf{B}$ onto a plane tangent to the Earth's surface. B_H can be resolved into north and east components (B_N and B_E). B_V is the projection onto the vertical axis. D is measured clockwise from North and ranges from $0 \rightarrow 360^\circ$. I is measured positive down from the horizontal and ranges from $-90 \rightarrow +90^\circ$ (because field lines can also point out of the Earth). $\mathbf{M}$ or $\mathbf{H}$ can be substituted for $\mathbf{B}$ as needed.

of observations of the geomagnetic field. You need about 48 observations to estimate the coefficients out to about $L = 6$ reliably.

The gauss coefficients are determined by fitting Equations 2.5 and 2.1 to observations of the magnetic field made by magnetic observatories or satellite data for a particular time. The *International (or Definitive) Geomagnetic Reference Field* for a given time interval is an agreed upon set of values for a number of Gauss coefficients, and their time derivatives. IGRF (or DGRF) models and programs for calculating various components of the magnetic field are available on the Internet from the National Geophysical Data Center; the address is <http://www.ngdc.noaa.gov>.

In order to get a feel for the importance of the various gauss coefficients, take a look at Table 2.1 which has the gauss coefficients for the first six degrees from Olsen et al. (2000). The power at each degree is calculated by $R_l = \sum_m (l+1)[(g_l^m)^2 + (h_l^m)^2]$ (Lowes, 1974) and this is shown in Figure 2.4. It is clear that the lowest order terms (degree one) totally dominate the field constituting some 90% of the field. This is why the field can often be assumed to be equivalent to that created by a simple dipole at the center of the Earth.

It is also interesting to view the estimated geomagnetic elements from the IGRF for 1995. Using the values for a given reference field in Equations 2.1 and 2.5, we can calculate values of B , D and I at any location on Earth. Examples of maps made from such calculations using the IGRF for 1995 are shown in Figure 2.5. These maps demonstrate that the field is a complicated function of position on the surface of the Earth.

The intensity values in Figure 2.5 are in general highest at the poles ($\sim 60 \mu\text{T}$) and lowest near the equator ($\sim 30 \mu\text{T}$), but the contours are not straight lines parallel to latitude as they would be for a field generated strictly by a geocentric axial dipole (GAD) such as that shown in Figure 2.6. Similarly, a GAD would produce lines of inclination that vary in a regular way from -90° to $+90^\circ$

Table 2.1: International Geomagnetic Reference Field, 2000 from Olsen et al. (2000).

l	m	g (nT)	h (nT)	l	m	g (nT)	h (nT)
1	0	-29614.72	0	5	0	-216.82	0
1	1	-1727.74	5186.27	5	1	350.96	43.69
2	0	-2267.11	0	5	2	222.36	171.65
2	1	3071.78	-2477.69	5	3	-130.72	-133.88
2	2	1672.18	-457.63	5	4	-168.53	-40.26
3	0	1340.85	0	5	5	-11.83	106.79
3	1	-2289.53	-226.99	6	0	72.34	0
3	2	1252.52	296.26	6	1	67.76	-17.09
3	3	715.1	-492.28	6	2	73.98	63.84
4	0	935.35	0	6	3	-161.06	65.19
4	1	786.71	271.92	6	4	-5.13	-61.34
4	2	250.81	-231.63	6	5	17.03	1.33
4	3	-404.66	118.54	6	6	-90.53	44.11
4	4	109.91	-303.88				

at the poles, with 0° at the equator; the contours would parallel the lines of latitude. Although the general trend in inclination shown in Figure 2.5b is similar to this GAD model field, there is considerable structure to the lines, which again suggests that the field is not perfectly described by a geocentric bar magnet. If the field were simply that of a geocentric axial dipole (a GAD field), declination would be everywhere zero. This is clearly not the case, as is shown by the plots of declination in Figure 2.5c.

The beauty of using the geomagnetic potential field is that the vector field can be evaluated anywhere outside the source region. Figure 2.6a shows the lines of flux predicted from the 1980 IGRF within the mantle. From this we can see that the field becomes simpler and more dipolar as we move from the core mantle boundary to the surface.

Perhaps the most important result of spherical harmonic analysis for our purposes is that the field is dominated by the first order terms ($l = 1$) and the external contributions are very small. The first order terms can be thought of as geocentric dipoles that are aligned with three different axes: the spin axis (g_1^0) and two equatorial axes that intersect the equator at the Greenwich meridian (g_1^1) and at 90° East (h_1^1).

2.2 Geocentric Axial Dipole (GAD) and other poles

To first order, the field is very much like one that would be produced by a gigantic bar magnet located at the Earth's center and aligned with the spin axis. In Figure 2.6b, we show a cross section of the Earth with a dipolar magnetic field superimposed. If the field were actually that of a *geocentric axial dipole* (GAD), it would not matter which cross section we chose because such a field is rotationally symmetric about the axis going through the poles; in other words, the magnetic field lines would always point North. The angle between the field lines and the horizontal at the surface of the Earth (inclination I), however, would vary between zero at the equator and 90° at the poles. Moreover, the magnetic field lines would be more crowded together at the poles than

2.2. GEOCENTRIC AXIAL DIPOLE (GAD) AND OTHER POLES

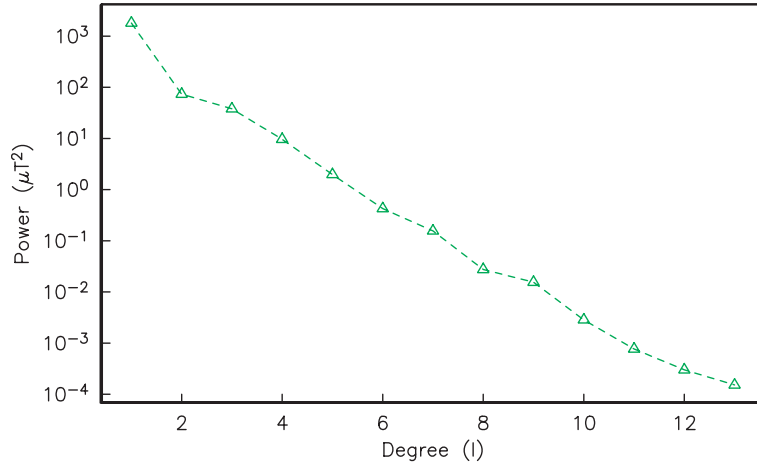


Figure 2.4: Power in of the geomagnetic field versus degree for the 2005 IGRF.

at the equator (the magnetic flux is higher at the poles) resulting in a polar field that would have twice the intensity of the equatorial field.

It turns out that when averaged over sufficient time, the geomagnetic field actually does seem on average to be that of a GAD field. This so-called *GAD model* of the field will serve as a useful crutch throughout our discussions of paleomagnetic data and applications.

The vector sum of the geocentric dipoles (g_1^0, h_1^0, h_1^1 in the IGRF) is a dipole that is currently inclined by 11° to the spin axis. The axis of this so-called *best-fitting dipole* pierces the surface of the Earth at the diamond in Figure 2.7. This point and its antipode are called *geomagnetic poles*. Points at which the field is vertical ($I = \pm 90^\circ$ shown by a triangle in Figure 2.7) are called *magnetic poles*, or sometimes, *dip poles*. These poles are distinguished from the *geographic poles* where the spin axis of the Earth intersects its surface. The Northern geographic pole is shown by a dot in Figure 2.7. Averaging ancient magnetic poles over some 10,000 years gives what is known as a *paleomagnetic pole*.

Because the geomagnetic field is axially dipolar to a first order approximation, we can write:

$$\psi_m = \frac{1}{\mu_0} a g_1^0 \left(\frac{a}{r} \right)^2 P_1^0(\cos \theta) = \frac{1}{\mu_0} a g_1^0 \left(\frac{a}{r} \right)^2 \cos \theta = \frac{1}{\mu_0} \frac{B_o \cos \theta}{r^2}, \quad (2.6)$$

where B_o is $g_1^0 a^3$. Note that this will give units in tesla if g_1^0 is given in tesla (as gauss coefficients normally are) Thus, from Equation 2.6,

$$B_N = \mu_0 H_N = \frac{B_o \sin \theta}{r^3}, \quad B_E = 0, \quad \text{and} \quad B_V = \mu_0 H_V = \frac{2B_o \cos \theta}{r^3}. \quad (2.7)$$

Given some latitude λ on the surface of the Earth in Figure 2.6 and using the equations for B_V and B_N , we find that:

$$\tan I = \frac{B_V}{B_N} = 2 \cot \theta = 2 \tan \lambda. \quad (2.8)$$

This equation is sometimes called the *dipole formula* which shows that the inclination of the magnetic field is directly related to the co-latitude (θ) for a field produced by a geocentric axial dipole

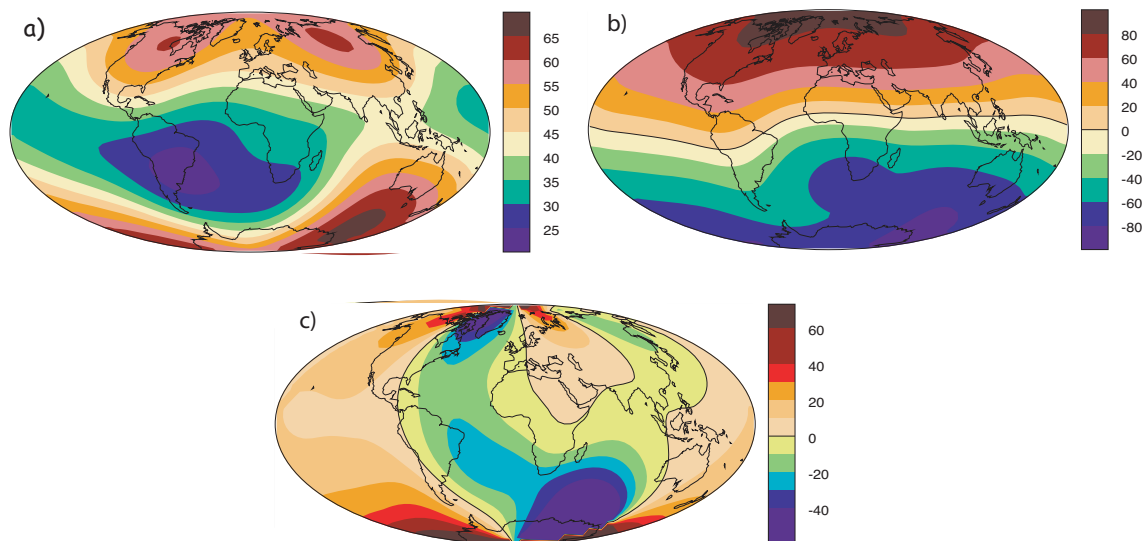


Figure 2.5: Maps of geomagnetic field of the IGRF for 1995 a) intensity in μT , b) inclination, c) declination.

(or g_1^0). The dipole formula allows us to calculate the latitude of the measuring position from the inclination of the (GAD) magnetic field, a result that is fundamental in plate tectonic reconstructions. The intensity of a dipolar magnetic field is also related to (co)latitude because:

$$B = (B_V^2 + B_N^2)^{\frac{1}{2}} = \frac{B_o}{r^3}(\sin^2 \theta + 4 \cos^2 \theta)^{\frac{1}{2}} = \frac{B_o}{r^3}(1 + 3 \cos^2 \theta)^{\frac{1}{2}}. \quad (2.9)$$

The dipole field intensity has changed by more than an order of magnitude in the past and the dipole relationship of intensity to latitude turns out to be not useful for tectonic reconstructions.

2.3 Plotting magnetic directional data

Magnetic field and magnetization directions can be visualized as unit vectors anchored at the center of a unit sphere. Such a unit sphere is difficult to represent on a 2-D page. There are several popular projections, including the Lambert equal area projection which we will be making extensive use of in later chapters. The principles of construction of the equal area projection are covered in the appendix.

In general, regions of equal area on the sphere project as equal area regions on this projection, as the name implies. Plotting directional data in this way enables rapid assessment of data scatter. A drawback of this projection is that circles on the surface of a sphere project as ellipses. Also, because we have projected a vector onto a unit sphere, we have lost information concerning the magnitude of the vector. Finally, lower and upper hemisphere projections must be distinguished with different symbols. The paleomagnetic convention is: lower hemisphere projections use solid symbols, while upper hemisphere projections are open.

The dipole formula assumes that the magnetic field is exactly axial. Because there are more terms in the geomagnetic potential than just g_1^0 , we know that this is not true. Because of the non-axial geocentric dipole terms, a given measurement of I will yield an equivalent *magnetic co-latitude* θ_m :

2.3. PLOTTING MAGNETIC DIRECTIONAL DATA

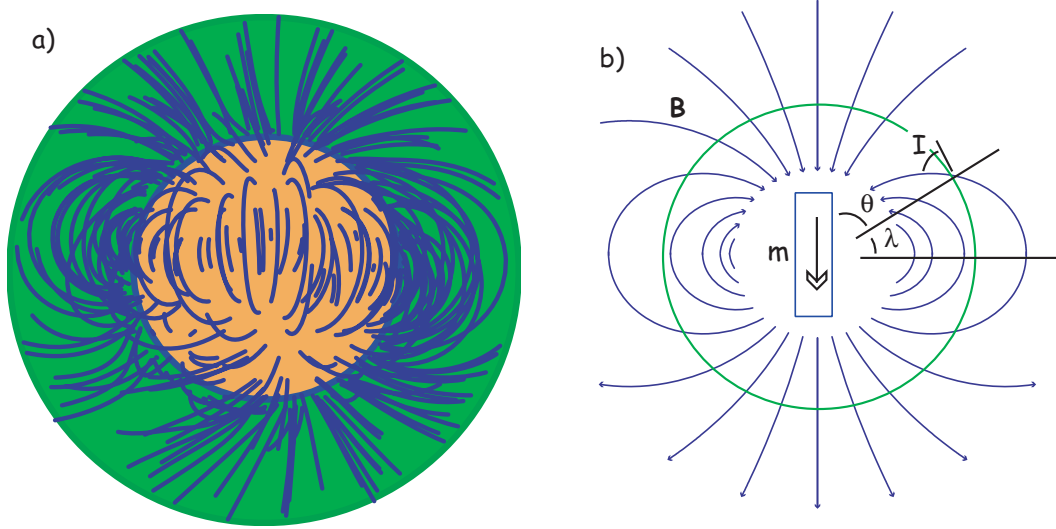


Figure 2.6: a) Predicted lines of flux for the 1980 IGRF. [Figure redrawn from R.L. Parker.] b) Orientations of magnetic field lines produced by a geocentric axial dipole m . λ and θ are latitude and co-latitude respectively. I is the local inclination at a particular point on the surface of the Earth.

$$\cot \theta_m = \frac{1}{2} \tan I. \quad (2.10)$$

Paleomagnetists often assume that θ_m is a reasonable estimate of θ and the validity of this assumption depends on several factors. We consider first what would happen if we took random measurements of the Earth's present field (see Figure 2.8). We randomly selected 200 positions on the globe (shown in Figure 2.8a) and evaluate the direction of the magnetic field at each site using the IGRF for 1995. These directions are plotted in Figure 2.8b using the paleomagnetic convention of open symbols pointing up and closed symbols pointing down. We also plot the inclinations as a function of latitude on Figure 2.8c. We see that, as expected from a predominantly dipolar field, inclinations cluster around the values expected for a geocentric axial dipolar field but there is considerable scatter and interestingly the scatter is larger in the southern hemisphere than in the northern one.

2.3.1 D', I' transformation

Often we wish to compare directions from distant parts of the globe. There is an inherent difficulty in doing so because of the large variability in inclination with latitude. In such cases it is appropriate to consider the data relative to the expected direction (from GAD) at each sampling site. For this purpose, it is useful to use the transformation proposed by (Hoffman 1984), whereby each direction is rotated such that the direction expected from a geocentric axial dipole field (GAD) at the sampling site is the center of the equal area projection. This is accomplished as follows:

Each direction is converted to cartesian coordinates (x_i) by:



Figure 2.7: The different magnetic poles. The triangle is the magnetic North Pole, where the magnetic field is straight down ($I = +90^\circ$). The diamond is the geomagnetic North Pole, where the axis of the best fitting dipole pierces the surface. The dot is the geographic North Pole. The dashed line is the magnetic equator where $I = 0^\circ$.

$$\begin{aligned}x_1 &= \cos D \cos I \\x_2 &= \sin D \cos I \\x_3 &= \sin I\end{aligned}$$

These are rotated to the new coordinate system (x'_i , see appendix to Lecture 1) by:

$$\begin{aligned}x'_1 &= (x_1^2 + x_3^2)^{1/2} \sin(I_d - \theta) \\x'_2 &= x_2 \\x'_3 &= (x_1^2 + x_3^2)^{1/2} \cos(I_d - \theta)\end{aligned}$$

where I_d = the inclination expected from a GAD field ($\tan I_d = 2 \tan \lambda$), λ is the site latitude, and θ is the inclination of the paleofield vector projected onto the N-S plane ($\theta = \tan^{-1}(x_3/x_1)$). The x'_i are then converted to D', I' by Equation 2.4. We see what happens to the directions shown in Figure 2.8b after the D', I' transformation in Figure 2.9. The latitudinal dependence of the inclinations has been removed. Yet, it is a feature of the geomagnetic field that directions from a given latitude tend to be more scattered in the up-down directions than from side to side. This tendency for elongation in the directions is most extreme at the equator and least at the poles, where directions are more or less circularly symmetric.

2.3.2 Virtual Geomagnetic Poles

We are often interested in whether the geomagnetic pole has changed, or whether a particular piece of crust has rotated with respect to the geomagnetic pole. Yet, what we observe at a particular

2.3. PLOTTING MAGNETIC DIRECTIONAL DATA

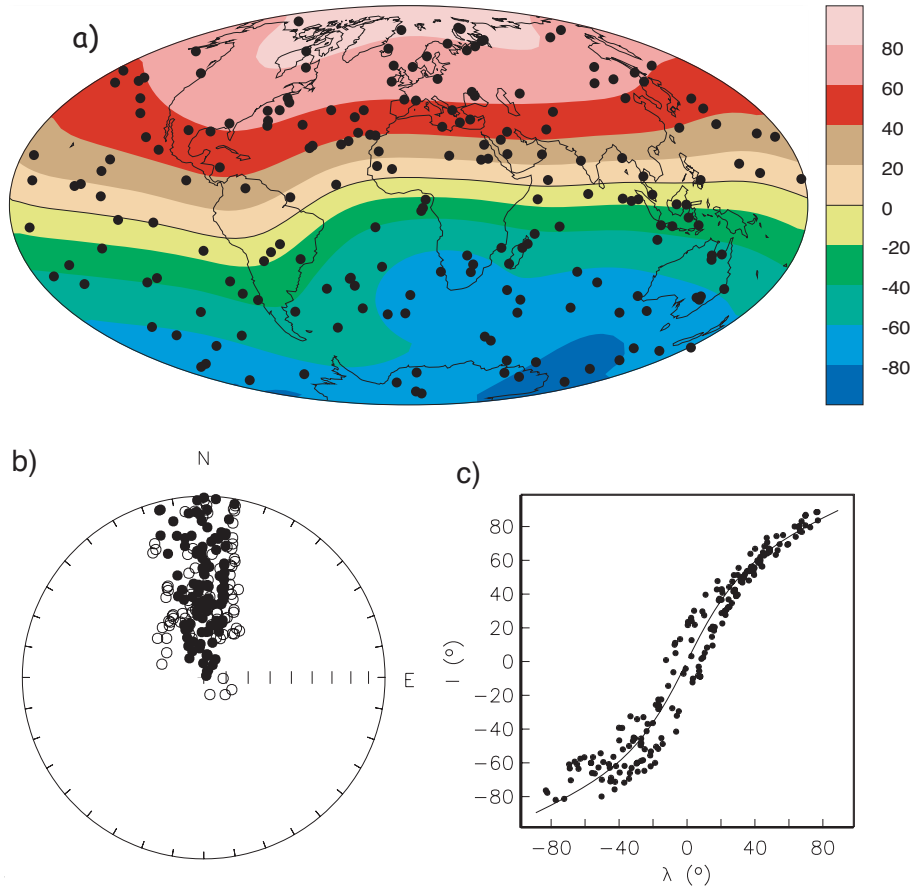


Figure 2.8: a) Hammer projection of 200 randomly selected locations around the globe. b) Equal area projection of directions of Earth's magnetic field as given by the IGRF evaluated for the year 1995 at locations shown in a). Open (closed) symbols indicate upper (lower) hemisphere. c) Inclinations (I) plotted as a function of site latitude (λ). The solid line is the inclination expected from the dipole formula (see text). Negative latitudes are south and negative inclinations are up.

location is the local direction of the field vector. Thus, we need a way to transform an observed direction into the equivalent geomagnetic pole.

In order to remove the dependence of direction merely on position on the globe, we imagine a geocentric dipole which would give rise to the observed magnetic field direction at a given latitude (λ) and longitude (ϕ). The *virtual geomagnetic pole* (VGP) is the point on the globe that corresponds to the geomagnetic pole of this imaginary dipole (Figure 2.10).

Paleomagnetists use the following conventions: ϕ is measured positive eastward from the Greenwich meridian and goes from $0 \rightarrow 360^\circ$. θ is measured from the North pole and goes from $0 \rightarrow 180^\circ$. Of course θ relates to latitude, λ by $\theta = 90 - \lambda$. θ_m is the magnetic co-latitude and is given by equation 2.10. Be sure not to confuse latitudes and co-latitudes. Also, be careful with declination. Declinations between 180° and 360° are equivalent to $D - 360^\circ$ and are counter-clockwise with respect to North.

The first step in the problem of calculating a VGP is to determine the magnetic co-latitude θ_m by equation 2.10 which is of course the dipole formula. The declination D is the angle from the

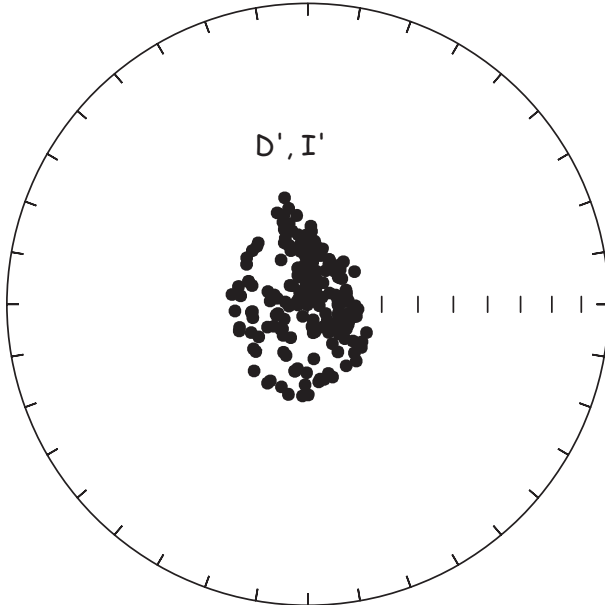


Figure 2.9: Directions from the Figure 13b transformed using the D', I' transformation.

geographic North Pole to the great circle joining S and P , and $\Delta\phi$ is the difference in longitudes between P and S , $\phi_p - \phi_s$. Now we use some tricks from spherical trigonometry as reviewed in the Appendix.

We can locate VGPs using the law of sines and the law of cosines. The declination D is the angle from the geographic North Pole to the great circle joining S and P (see Figure 2.10) so:

$$\cos \theta_p = \cos \theta_s \cos \theta_m + \sin \theta_s \sin \theta_m \cos D, \quad (2.11)$$

which allows us to calculate the VGP co-latitude θ_p . The VGP latitude is given by:

$$\lambda_p = 90 - \theta_p,$$

so $90 > \lambda_p > 0$ in the northern hemisphere and $0 < \lambda_p < 90$ in the southern hemisphere.

To determine ϕ_p , we first calculate the angular difference between the pole and site longitude $\Delta\phi$.

$$\sin \Delta\phi = \sin \theta_m \cdot \frac{\sin D}{\sin \theta_p}. \quad (2.12)$$

If $\cos \theta_m \geq \cos \theta_s \cos \theta_p$, then $\phi_p = \phi_s + \Delta\phi$. However, if $\cos \theta_m < \cos \theta_s \cos \theta_p$ then $\phi_p = \phi_s + 180 - \Delta\phi$.

Now we can convert the directions in Figure 2.8b to VGPs (Figure 2.11). The grouping of points is much tighter in Figure 2.11 than in the equal area projection because the effect of latitude variation in dipole fields has been removed.

If a number of VGPs are averaged together, the average pole position is called a “paleomagnetic pole”. How to average poles and directions is the subject of another lecture, however.

A. EQUAL AREA PROJECTIONS

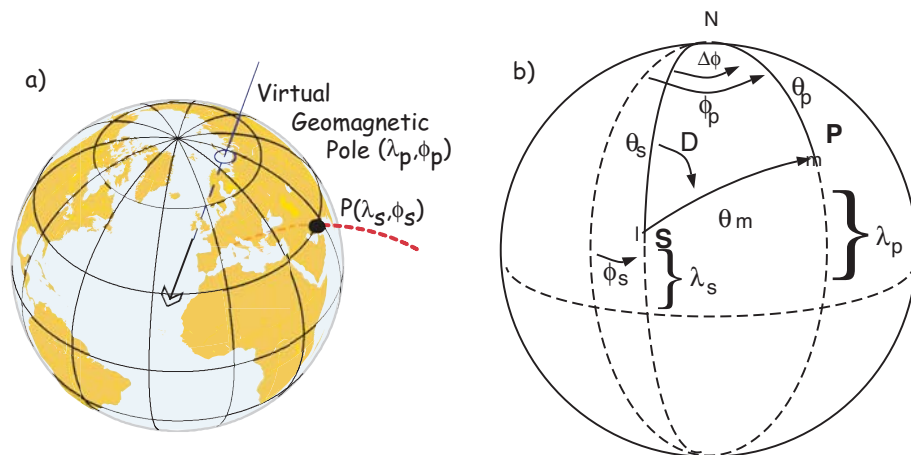


Figure 2.10: Transformation of a direction measured at S into a virtual geomagnetic pole position P, using principles of spherical trigonometry and the dipole formula. a) Illustration of the magnetic field line observed at position P and its associated VGP. b) More detailed view. Site S has latitude λ_s and longitude ϕ_s and a magnetic field direction D and I . The co-latitude of S is θ_s . $\Delta\phi$ is the difference in longitude between S and P and θ_p is the co-latitude of P. The direction can be transformed to an equivalent VGP (at P) with latitude λ_p and longitude ϕ_p as described in the text. The co-latitude of S with respect to P is the *magnetic co-latitude* θ_m . N is the geographic North Pole (the spin axis of the Earth).

2.3.3 Virtual Dipole Moment

As pointed out earlier, magnetic intensity varies over the globe in a similar manner as inclination. It is often convenient to express paleointensity values in terms of the equivalent geocentric dipole moment which would have produced the observed intensity at that (paleo)latitude. Such an equivalent moment is called the *virtual dipole moment* (VDM) by analogy to the VGP. First, the magnetic (paleo)co-latitude θ_m is calculated as before from the observed inclination and the dipole formula of equation 2.8, then following the derivation of equation 2.9,

$$\text{VDM} = \frac{4\pi r^3}{\mu_o} B_{\text{ancient}} (1 + 3 \cos^2 \theta_m)^{-\frac{1}{2}}. \quad (2.13)$$

Sometimes the site co-latitude as opposed to magnetic co-latitude is used in the above equation, giving a *virtual axial dipole moment* (VADM).

Appendix

In this appendix we will review the basic techniques necessary useful for understanding Lecture 2. In particular, we will cover plotting of equal area projections and spherical trigonometry.

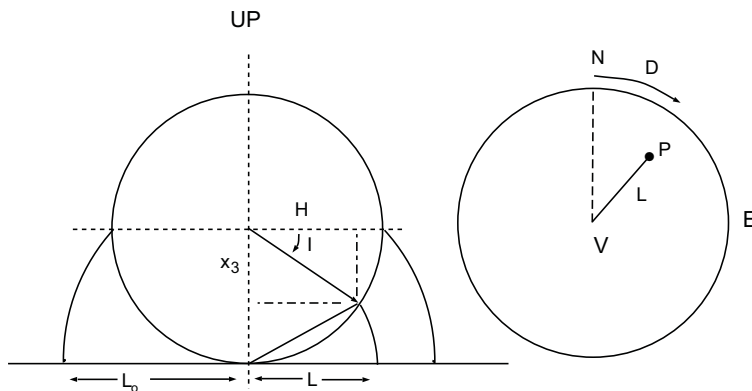
A Equal area projections

The principles for how to make an equal area projection are shown in Figure A1. The point P corresponds to a D of 40° and I of 35° . D is measured around the perimeter of the equal area net

and I is transformed as follows:

where $L_o = 1/\sqrt{x_1^2 + x_2^2}$.

In Figure B1, α, β and γ are the angles between the great circles labelled a, b , and c . On a unit sphere, a, b and c are also the angles subtended by radii that intersect the globe at the apices A, B, and C (see inset on Figure B1). Two formulae from spherical trigonometry come in handy in



Lectures in Paleomagnetism

B. SPHERICAL TRIGONOMETRY

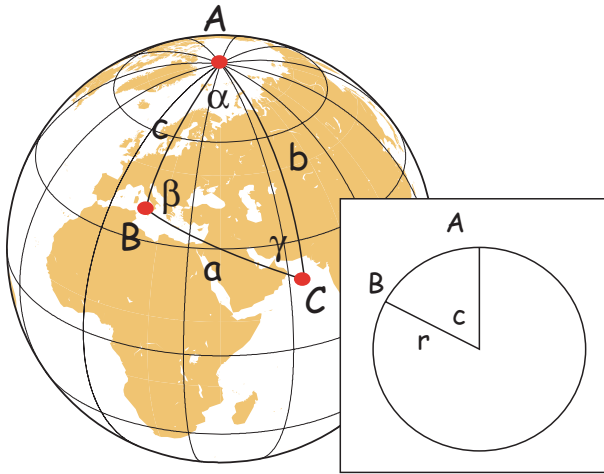


Figure B1: Rules of spherical trigonometry. a, b, c are all great circle tracks on a sphere which form a triangle with apices A, B, C . The lengths of a, b, c on a unit sphere are equal to the angles subtended by radii that intersect the globe at the apices, as shown in the inset. α, β, γ are the angles between the great circles.

paleomagnetism, the Law of Sines:

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}, \quad (\text{B1})$$

and the Law of Cosines:

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha. \quad (\text{B2})$$

Bibliography

- Butler, R. F. (1992), *Paleomagnetism: Magnetic Domains to Geologic Terranes*, Blackwell Scientific Publications.
- Hoffman, K. (1984), ‘A method for the display and analysis of transitional paleomagnetic data’, *J. Geophys. Res.* **89**, 6285–6292.
- Lowes, F. (1974), ‘Spatial power spectrum of the main geomagnetic field and extrapolation to the core’, *Geophys. J. R. Astron. Soc.* **36**, 717–730.
- Merrill, R. T., McElhinny, M. W. & McFadden, P. L. (1996), *The Magnetic Field of the Earth: Paleomagnetism, the Core, and the Deep Mantle*, Academic Press.
- Olsen, N., Sabaka, T. & Toffner-Clausen, L. (2000), ‘Determination of the IGRF 2000 model’, *Earth Planet. Sci. Lett.* **52**, 1175–1182.
- Tauxe, L. (2005), ‘Inclination flattening and the geocentric axial dipole hypothesis’, *Earth Planet. Sci. Lett.* **233**, 247–261.