Surface waves (continuing)

For evanescent waves such as Rayleigh and Love waves we have seen that long wavelength waves penetrate deeper into the half space than short-wavelength waves.
How exactly structure in a certain depth interval influences a wave of a particular frequency is described by a **sensitivity kernel**. They represent the maximum particle motion at a certain depth as a function of frequency, which can be computed from a reference Earth model.

Excitation of surface waves

Figure 1 can be used to understand in qualitative sense the excitation of surface waves by earthquakes. In general, the position of the earthquake (i.e. the depth in our case of depth-dependent media) determines which modes can be excited. A **fundamental mode** has no displacement deeper than a certain depth; by reciprocity, a source (assume a white spectrum of the source so that it can — in principle — excite all frequencies) that is located at those large depths will not cause displacement of that fundamental mode at the surface.

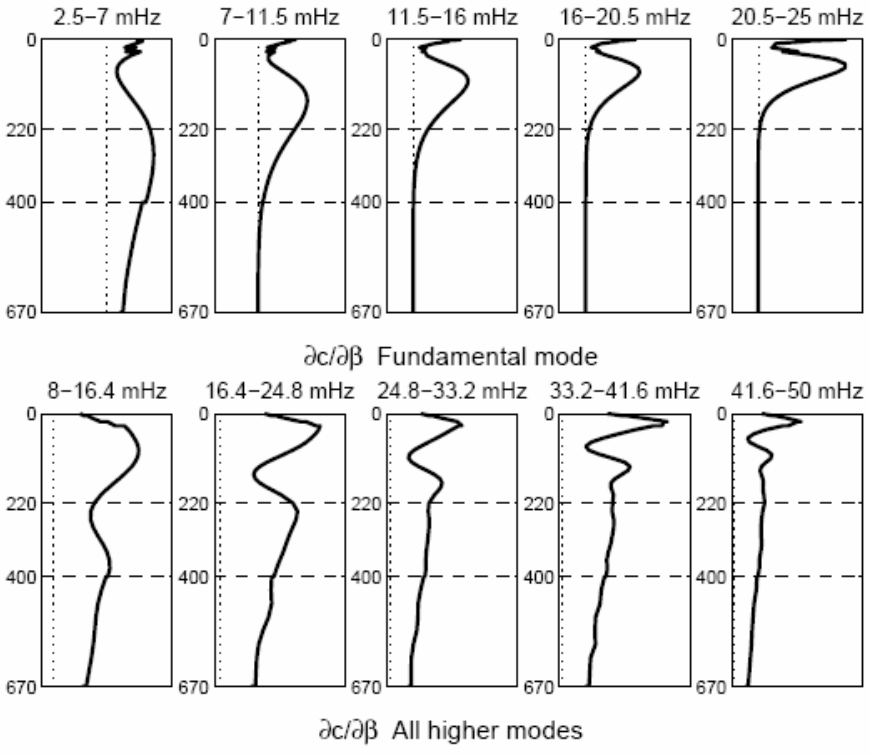


Figure 1:phase speed sensitivity kernels

Dispersion curves

We have seen that the radial variation of shear wave speed causes dispersion of the surface waves. This means that the observed surface wave dispersion contains structural information about the radial variation of seismic properties. A plot of the group or phase velocity as a function of frequency is called a **dispersion curve**. Their diagnostic value of 1D structure has been explored in great detail. Typically, the curves produced from observed records are matched with standard curves computed from an assumed reference Earth model that can have a structure that is characteristic for a certain type of upper mantle (e.g., old/young continents, old/young oceans, etc.). Such analyses have produced the first maps of the thickness of oceanic lithosphere which revealed the increase in thickness with increasing age of the lithosphere (or distance from the ridge. Figure 2 shows a variety of typical dispersion curves for different tectonic provinces.

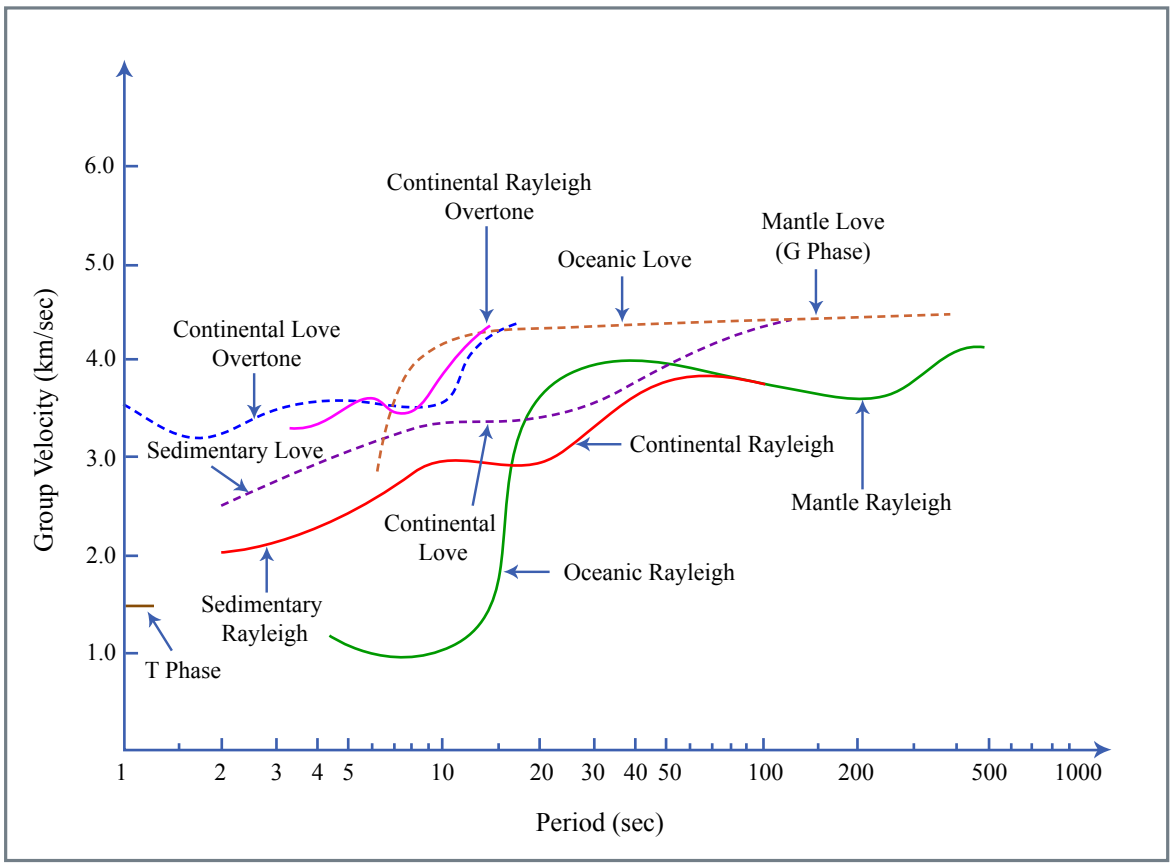


Figure by MIT OCW.

Figure 2

In Figure 3, the dashed lines are travel time curves for those S/SS/SSS phases. But note that the frequency of those phases change with distance, so that the waveform changes. For instance, with increasing distance, the first arriving phase is composed of waves with larger frequencies (because they sample deeper).

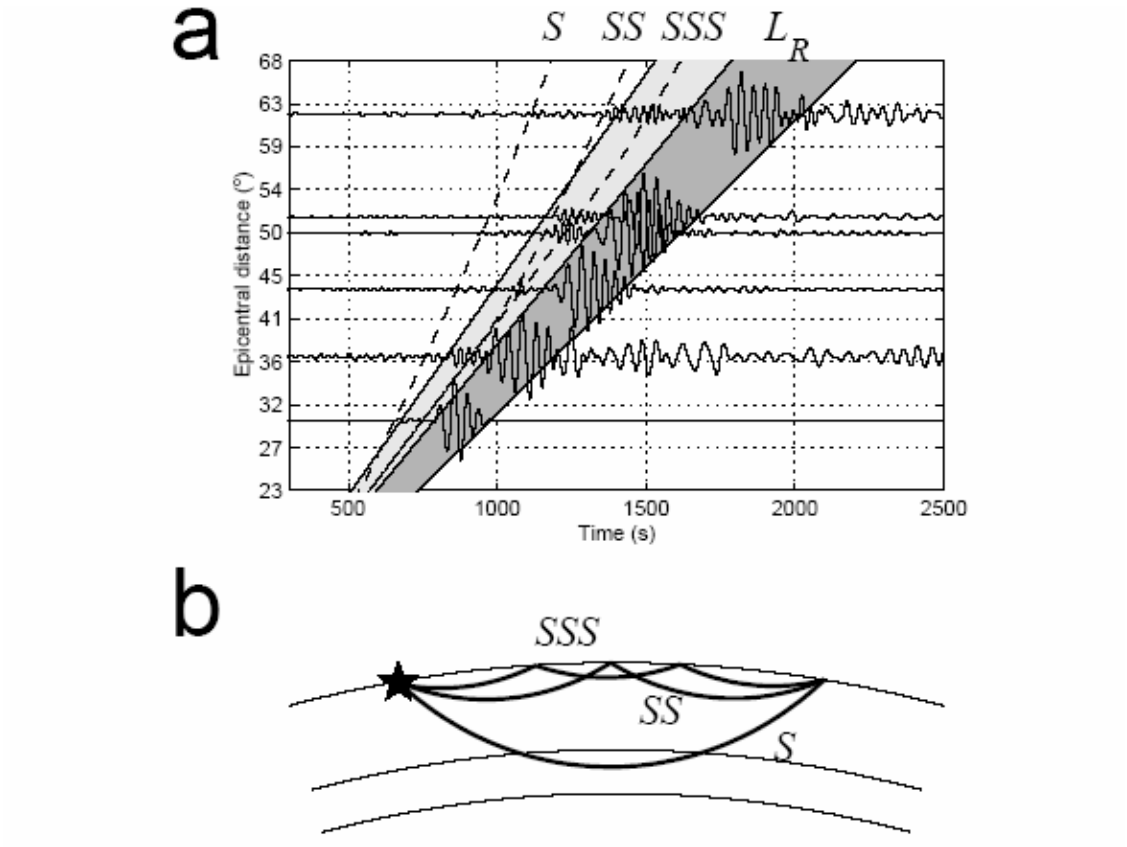


Figure 3: group velocity windows and phase velocity curves

The group velocity of surface waves of a particular frequency defines a straight line through the origin and through the signal of that particular frequency on records of ground motion at different distances. The group velocity decreases as the frequency increases. As a result, high frequency phases become less and less pronounced with increasing distance from the source (or time in the seismogram). The group velocity is very important: the energy in surface waves propagates mainly in the constructively interfering wave packets, which move with the group velocity. Narrow-band filtering can isolate the wave packets with specific central frequencies (see Figure 4), and the group velocity for that frequency can then be determined by simply dividing the path length along the surface by the observed travel time. This technique can be used for the construction of dispersion curves (figure 2).

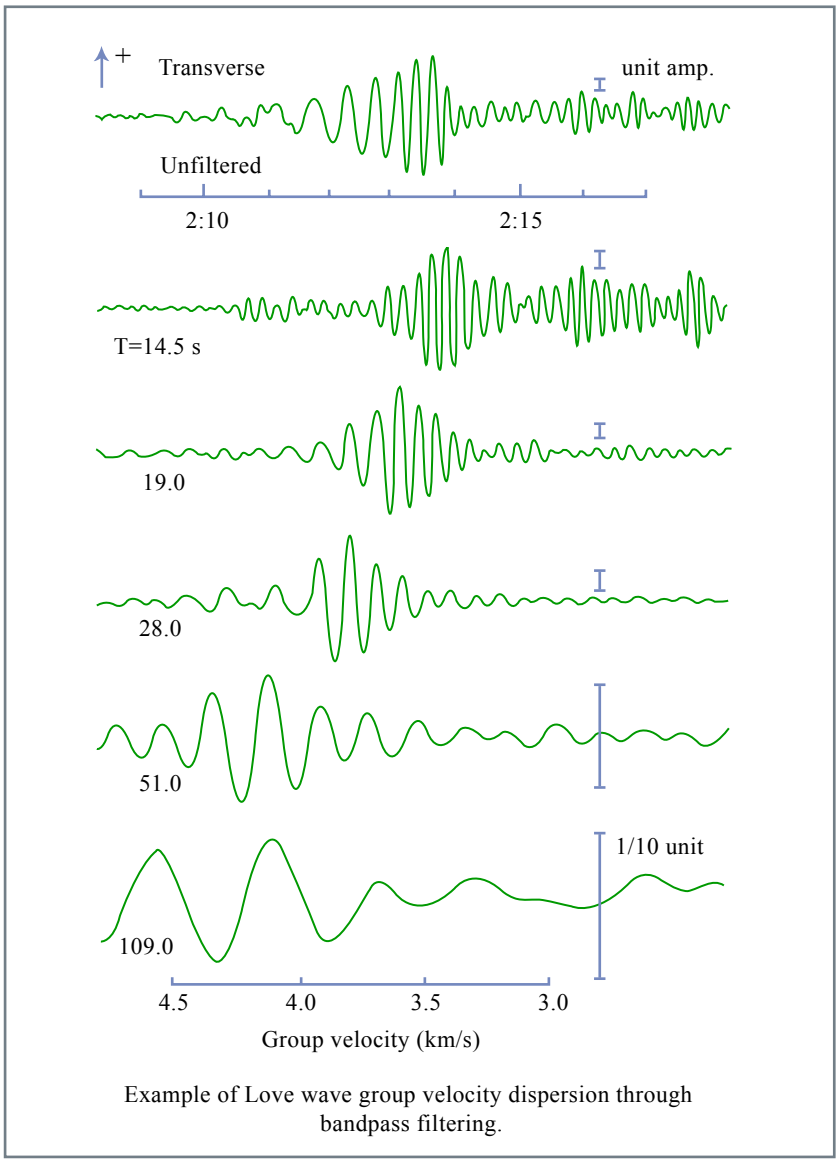


Figure by MIT OCW.

Figure 4: group velocity dispersion through bandpass filtering

Surface wave Tomography

- Recall from the travel time tomography:

projector (consists of ray segments)

$$\underline{d} = \underline{\underline{A}}\,\underline{m}$$

data — model

observations $\delta t = T - T_o$ — **model perturbation ($m-m_o$)**

$$d_i = \int_{volume} G_i(\underline{r})\,\mu(\underline{r})\,d\underline{r}$$

sensitivity kernel/ Frechet derivatives

$$\delta t = \int \Delta s.dl$$ Integration over the ray path

Surface wave tomography:

- $$\underline{d}' = \underline{\underline{A}}' \underline{m}$$

We have the same model as for body wave tomography.

Thus, we can combine the two:

$$\begin{pmatrix} d \\ d' \end{pmatrix} = \begin{pmatrix} A \\ A' \end{pmatrix} \underline{m}$$

- When we do surface wave tomography, we measure the phase (or group) velocity: δc_i for an earthquake i.

- As with the body wave tomography, we will look at **perturbations between the model and the data**:

$$\delta c_i(\omega) = \frac{1}{\Delta} \int_{path} \delta c(\omega, \theta, \varphi) dl$$

Ray path

- $\underline{d}' = \underline{\underline{A}}' \underline{m}$

Phase velocity perturbation model at a position δc (ω,θ,φ)

→ We can construct **phase velocity maps**

- Those phase velocity maps are first constructed for a particular **frequency**. But, because of the evanescence of the surface waves, we can see what frequency is sensitive to a particular depth.
→ convert ω to z

To do so, we look at the sensitivity kernel for different frequencies (figure 1). We use $\frac{\partial c}{\partial \beta}$ and we get:

$$\delta c_i(\omega) = \frac{1}{\Delta} \int_{path} \left[\left(\frac{\partial c}{\partial \beta} \right)_{\omega, z} \delta\beta(\theta, \varphi, z) dz \right] dl$$

$$\partial c = \frac{\delta c}{\delta \beta} \partial \beta$$

The unknown here is: $\delta\beta(\theta,\varphi,z)dz$ = m 3D model of β

- So, using the decay of the amplitudes with z, we can talk about a certain z and not only a frequency.
  Nevertheless,
  - phase velocity map *at a certain frequency*: more robust
  - phase velocity map *at a certain depth*: make an assumption, the model depends on the sensitivity kernel.

- $\left(\frac{\partial c}{\partial \beta}\right)_z$: 1D Earth model, with an average velocity for each depth
- Only data from the top upper mantle (<660 km) can be used. Phases that sample too deep must be excluded. To do so, windows are applied on seismograms in order to drop phases with too high phase velocity.
  → Select phases in order to use 1D earth model

- If Rayleigh waves are used, c depends not only on β but on α (and also on ρ)!

$$\partial c = \frac{\delta c}{\delta \beta}\partial\beta + \frac{\delta c}{\delta \alpha}\partial\alpha + \frac{\delta c}{\delta \rho}\partial\rho$$

The sensitivity kernel is not zero for α and ρ, especially at shallow depths but the one of β overwhelms the 2 others. BUT, still a strong approximation.

Waveform modeling

1. Non linear waveform inversion:
   Try to find the best-fitting model → end up with series of 1D model β(z)
2. Construct 3D model for all 1D models

An example of waveform modeling:

Image removed due to copyright considerations.

Please see:

Maggi, Alessia, and Keith Priestley. "Surface Waveform Tomography of the Turkish-Iranian Plateau." *Geophysical Journal International* 160, no. 3: 1068-1080. doi: 10.1111/j.1365-246X.2005.02505.x.

Figure 5: Sensitivity of 1-D waveform inversions to focal depths. The effects of varying focal depth for two events: (a) focal depth 2 km (b) focal depth 9 km. Fits of final synthetic (dashed) to observed (solid) seismograms for shifts in focal depths of 10-50 km are shown on the left and the corresponding inversion velocity models (thin lines) are shown on the right along with the velocity model for the correct depth (thick line). The misfit and maximum frequency achieved by the waveform inversion are denoted to the right of each waveform fit.