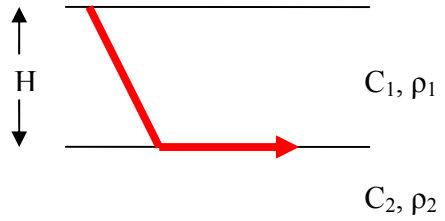
Quick review from last time:



Ground roll – Love waves

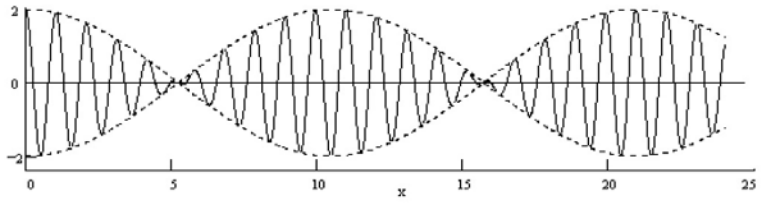
w, k → k = w/c

$$\tan\left\{wH\left(\frac{1}{c_1}\right)^2 - p_n\right\}^{1/2} = \frac{\rho_2\left(\left(\frac{1}{c_1}\right) - p_n\right)^{1/2}}{\rho_1\left(p_n - \left(\frac{1}{c_2}\right)^2\right)^{1/2}}$$, where n is the model #

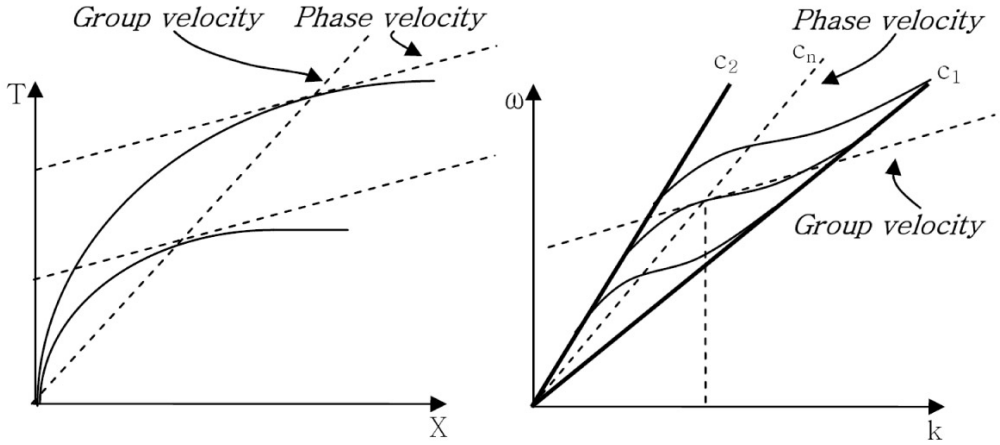
in the Layer: oscillatory wave → x-direction
in the half space: evanescent wave → x-direction

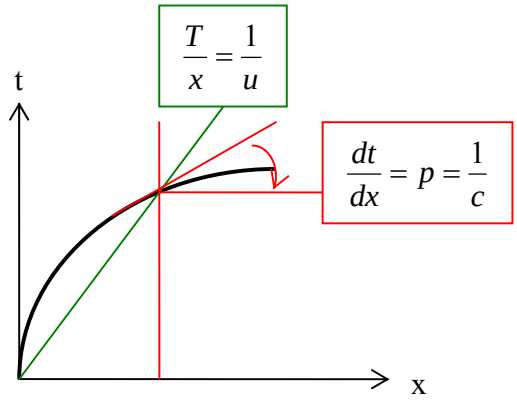
$$\frac{1}{c_2} < p < \frac{1}{c_1}$$

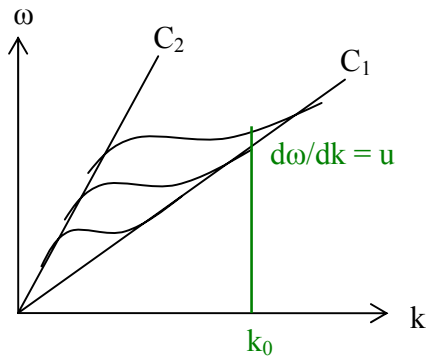
phase velocity c = ω/k group velocity u = dω/dk = x/t = distance/time



The dotted line represents the group velocity and the solid line represents the phase velocity.







$\lim_{\omega\to\infty} u = \lim_{\omega\to\infty} \frac{d\omega}{dk} = \frac{\omega}{k} = c_1$

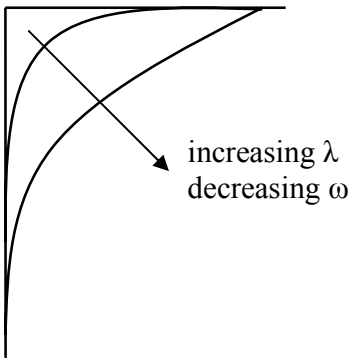
$\therefore \lim_{\omega\to\infty} u = c_1$

Arrival time:
$T = x/c_1$ $\quad$ $u = x/T$

$$u = \frac{d\omega}{dk} \qquad \rightarrow \qquad u = c + k\frac{dc}{dk} \rightarrow k = \frac{2\pi}{\lambda}$$

$$k = \frac{\omega}{c} \rightarrow \omega = ck \qquad \therefore u = c - \lambda\frac{dc}{d\lambda}$$

Can look at the evanescence of the wave…

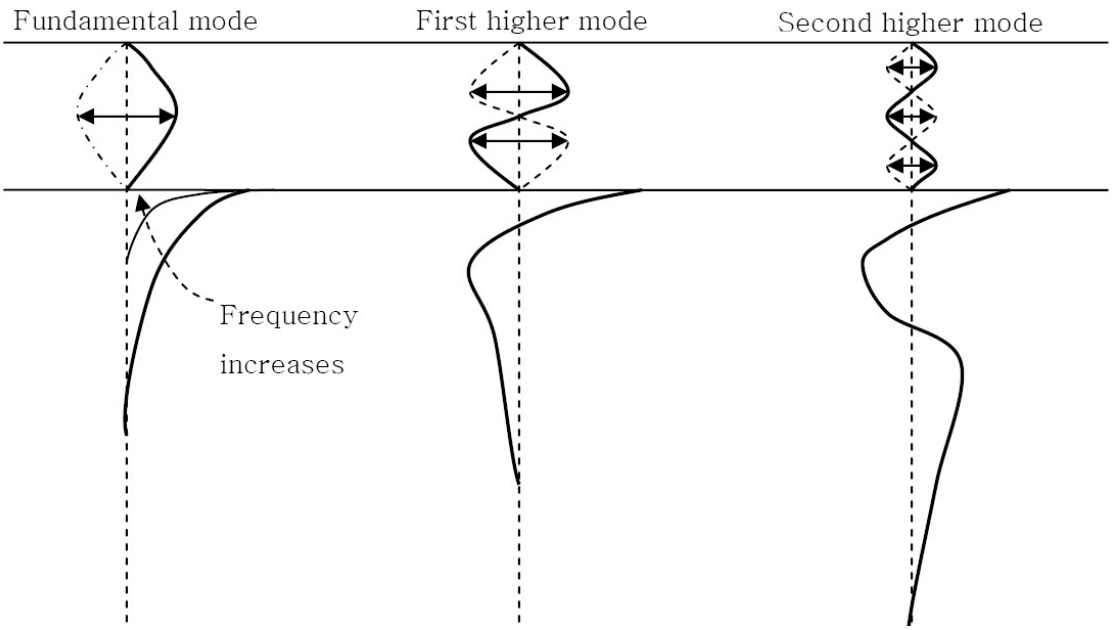


$e^{-\eta wz}$

$e^{-k_z z} = e^{-\frac{2\pi}{\lambda}t}$

*low frequency wave is more sensitive to deep structure. Therefore, low frequency wave should arrive earlier than high frequency wave.

Just looking at the fundamental mode will give us some information about shallow depths. Combing with higher modes will give even more information about what is happening at depth.



The higher the mode the better the sensitivity of deeper structure. There are distinct modes, which are understood through interference boundary conditions, thus constraining which combinations can be used. Some of the modes will not propagate as a wave because there is no constructive interference.

Principle of the stationary phase: Only certain frequencies and directions will interfere constructively to create arrivals. If at a particular time you have wave propagation they add up to give the seismogram amplitude.

Seismogram: $u(x,t) = \int_{\omega k} A(\omega,k) e^{i(kx-\omega t)} d\omega dk$ ← plane wave superposition

Integrate together to give the displacements…
Building interference in means many combinations of ω & k will not result in displacement.

$$u(x,t) = \int_{\omega k} A(\omega,k) e^{i(kx-\omega t)} d\omega dk$$

$$\frac{d}{d\omega}(kx-\omega t) = 0 \quad \rightarrow \quad \text{expressions of stationary phase}$$

$$\frac{d}{dk}(kx-\omega t) = 0$$

$$\frac{d\omega}{dk} = \frac{x}{t} = u \text{ group velocity}$$

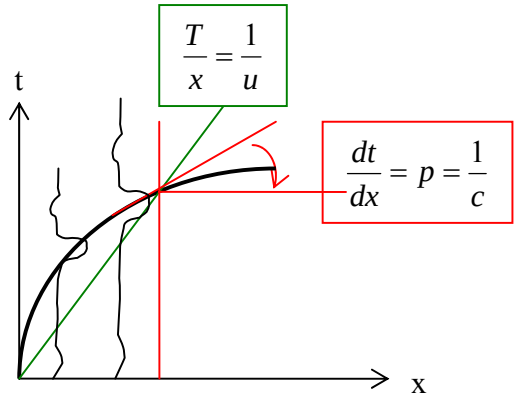
* has a beating affect – interference of waves. Highest amplitude where many arrive at the same time, and the other ω's & k's exist, but do not produce the amplitude.

In S & W: Read 2.7 and 2.8
2.8.2 does not call it the stationary phase, but they look at f(ω,k)=0, which is the stationary phase approach.

$$\frac{d\left(\omega\left(\frac{x}{c}-t\right)\right)}{d\omega} \rightarrow u = \frac{c}{\left(1-\frac{\omega}{c}\frac{dc}{d\omega}\right)}$$ as a consequence of stationary phase principle

Airy Phase – wave that arises if the phase and the change in group velocity are stationary $\frac{du}{d\omega} = 0$ → gives the highest amplitude in terms of group velocity and are prominent on the seismogram.

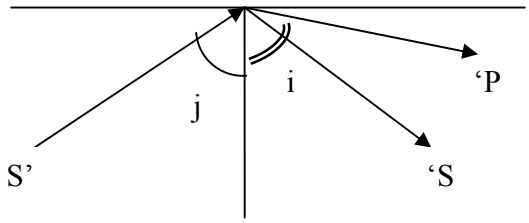
Surface arrival period T~ 20-30sec…You can use this to filter the surface waves from the seismogram.



Part of the wave that propagates with the group velocity is not same part of the seismogram. The peaks and troughs are related to the phase velocity (e.g. first onset). Group velocity is related to frequency band.

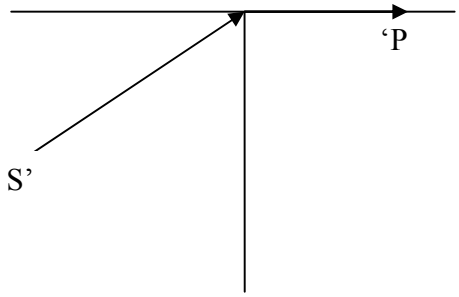
With greater propagation distances the arrivals spread out more and shift to lower frequencies. Looking at fundamental and higher modes becomes easier at higher distances because they are more spread out.

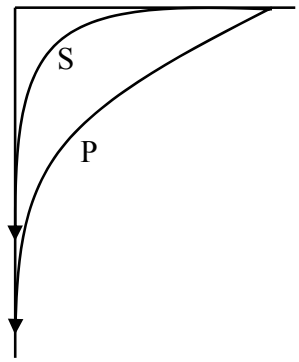
** *Rayleigh → more complicated than Love waves*



$$\frac{\sin j}{\beta} = \frac{\sin i}{\alpha}$$

if $\beta < \alpha$, can have critical reflection and horizontal propagating p-wave. If $j > j_c$ then there will be evanescence in the p-wave ($p > 1/\alpha$).





both p-wave and s-wave horizontal propagation if $p > \frac{1}{\beta} > \frac{1}{\alpha} = \frac{1}{c}$. If a wave comes in with a 1/c that is larger than local 1/α and 1/β, the above will occur. This will also happen if the source emits a horizontal energy (this is rare).

$$P : \phi = A\exp\{-\omega\hat{\eta}_\alpha z\}\exp[i\omega(px-t)] \rightarrow \eta_\alpha = \sqrt{\frac{1}{\alpha^2} - p^2} = i\hat{\eta}_\alpha, p > \frac{1}{\alpha}$$

$$S_v : \Psi_{sv} = B\exp[-\omega\hat{\eta}_\beta z]\exp[i\omega(px-t)] \rightarrow \eta_\beta = \sqrt{\frac{1}{\beta^2} - p^2} = i\hat{\eta}_\beta, p > \frac{1}{\beta}$$

Boundary conditions: Kinematic and dynamic

$u = \Delta\phi + \Delta \times \psi$

Zoeppritz:

$$\begin{bmatrix} (\lambda+2\mu)\eta_\alpha^2 + \lambda p^2 & 2\mu p\eta_\beta \\ 2p\eta_\alpha & p^2 - \eta_\beta^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Trivial solution is A=B=0.

Non-trivial solution leads to:

$$\left[(\lambda+2\mu)\eta_\alpha^2 + \lambda p^2\right]\left(p^2 - \eta_\beta^2\right) - 2\rho\eta_\alpha\left(2\mu p\eta_\beta\right) = 0$$

This expression → a.k.a. Rayleigh wave denominator (Rayleigh, 1887) can be written looking at wave speeds, but usually done numerically assuming a Poisson's medium (λ=μ) and $\alpha = \sqrt{3}\beta$. This scaling will help to simplify the above equation.

$$\alpha = \left(\frac{\lambda + 2\mu}{\rho}\right)^{1/2}$$

$$\beta = \left(\frac{\mu}{\rho}\right)^{1/2}$$

→ $\alpha = \sqrt{3}\beta$

$$\boxed{\left(\frac{c^2}{\beta^2}\right)^3 - 8\left(\frac{c^2}{\beta^2}\right)^2 - \frac{56}{3}\left(\frac{c^2}{\beta^2}\right) - \frac{32}{3} = 0}$$

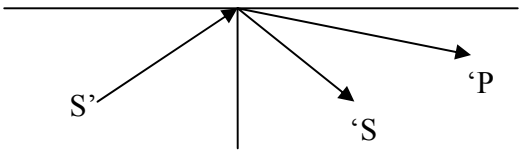
c_R = phase velocity of Rayleigh waves

Solutions

(1) $\left(\frac{c}{\beta}\right)^2 = u \rightarrow \frac{c}{\beta} = 2$ in this situation

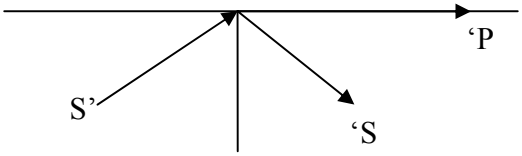
$$\eta_\alpha = \sqrt{\frac{1}{\alpha^2} - p^2} \in \Re$$

$$\eta_\beta = \sqrt{\frac{1}{\beta^2} - p^2} \in \Re$$

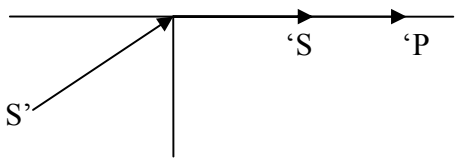


$$p < \frac{1}{\alpha} < \frac{1}{\beta}$$

(2) $\left(\frac{c}{\beta}\right)^2 = 2 + \frac{2}{3}\sqrt{3} \rightarrow \frac{1}{\alpha} < p < \frac{1}{\beta}$ in this situation



(3) $\left(\frac{c}{\beta}\right)^2 = 2 - \frac{2}{3}\sqrt{3} \rightarrow \frac{1}{\alpha} < \frac{1}{\beta} < p$ in this situation



$c_R \sim 0.92\beta$ which makes it about 10% slower than the shear wave velocity. This explains why the Rayleigh wave is slower than the love wave. The Love wave, at low frequency → c_2 and at high frequencies Love wave (at it's slowest) is c_1…Rayleigh wave is about 90% of the Love wave at the most.