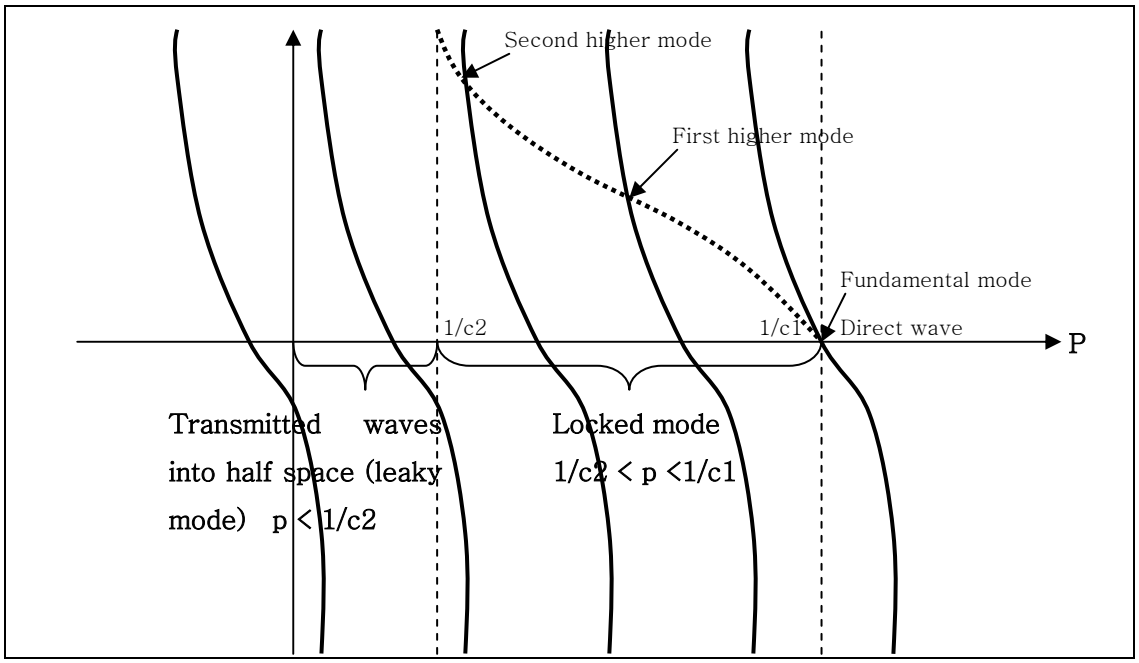
TODAY'S LECTURE

1. Review of ground roll/Love waves
2. Evanescent wave and sensitivity kernel
3. ω – k space
4. Phase velocity vs. group velocity

REVIEW OF GROUND ROLL/LOVE WAVES

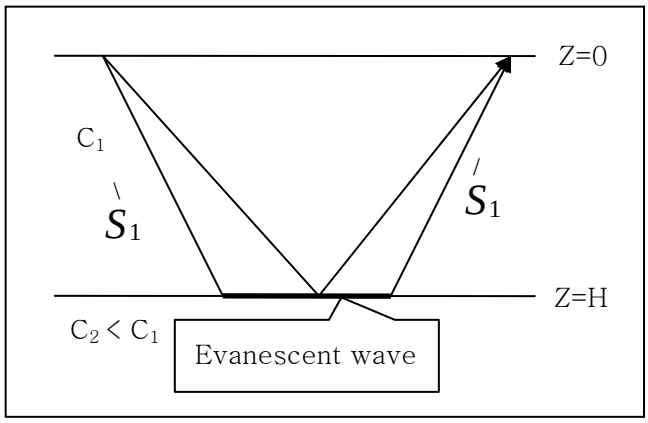
The dispersion relationship for ground roll is given by

$$\tan(\omega H\sqrt{\frac{1}{c_1^2}-p^2}) = \frac{\rho_2\sqrt{\frac{1}{c_1^2}-p^2}}{\rho_1\sqrt{p^2-\frac{1}{c_2^2}}} \qquad (1)$$



In the leaky mode (precritical), the waves can be transmitted into the half space. In the locked mode (postcritical), all of the energy stays in the upper layer. A finite number of modes have discrete phase velocities. The fundamental mode represents a direct wave and there can be higher modes depending on the wave frequency. It is because the left side of equation (1) includes the wave frequency. In the low frequency case, the spacing of the curves increases (the number of higher mode decrease and the fundamental mode can be exist only). On the other hand, the spacing decreases in the high frequency case (the number of higher mode increases. In the extreme case, it can be continuous.)

EVANESCENT WAVE AND SENSITIVITY KERNEL



Evanescent wave :

$$\rightarrow \exp\{-\omega\eta z\}\exp\{i(kx-\omega t)\}$$

The first term represents exponential decay with respect to z for the postcritical case.

In the layer (0<Z<H), the displacement is given by

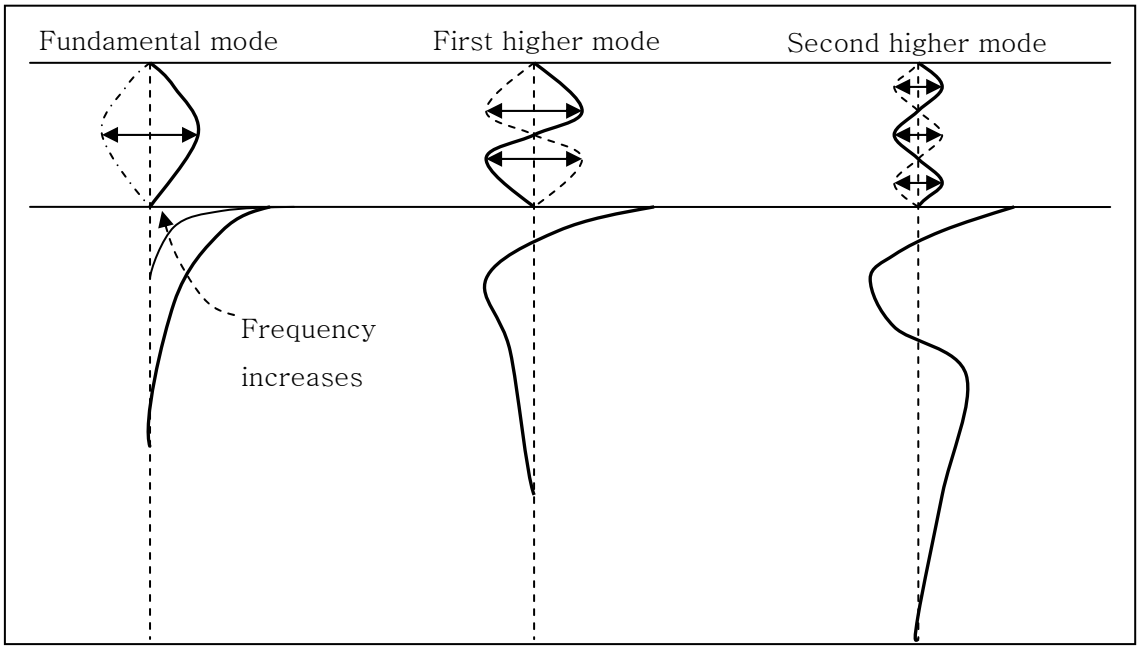
$$u_n(x,z,t) = 2\overset{\backslash}{S}_1 \cos\{\omega\sqrt{\frac{1}{c_1^2}-p_n^2}\,z\}\exp\{i(k_n x-\omega t)\} \qquad (2)$$

p_n can be obtained by the dispersion relation from equation (1). It represents the slowness of n[th] higher mode (overtone), $p_n = 1/c_n$. The exponential term of equation (2) represents wave propagation in x direction with wavenumber k_n and phase velocity $c_n = w/k_n$ in which max phase velocity c2 and min phase velocity c1. the displacement varies with depth as the cosine term of equation (2), and so oscillates. The cosine term depends on the wave frequency, higher mode slowness and depth.

In the half space (z>H), the displacement is given by

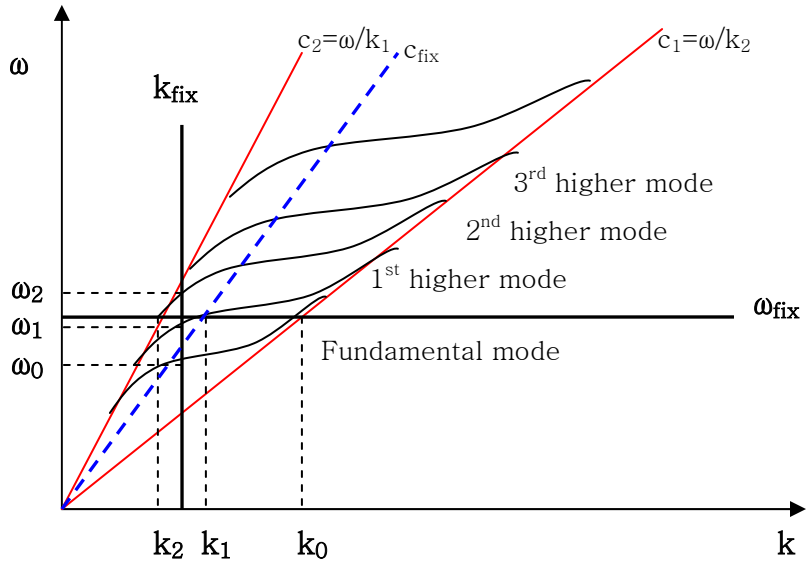
$$u_n(x,z,t) = 2\overset{\backslash}{S}_1 \cos\{\omega\sqrt{\frac{1}{c_1^2}-p_n^2}\,H)\}\exp\{-\omega\sqrt{p_n^2-\frac{1}{c_2^2}}(z-H)\}\exp\{i(k_n x-\omega t)\} \qquad (3)$$

As in the layer, we have a wave propagating in the x direction in the half space with wavenumber k_n . The displacement decays exponentially with depth as the first exponential term. In the low frequency case, the decay is slow and the amplitude is large at c=c2. In the high frequency case, the decay is fast. If the frequency is infinite, there is no sensitivity. Moreover, as the depth goes to infinity, the displacement or sensitivity is going to zero. The following figure describes the pattern of the wave in the layer and in the half space. We can see that higher modes sample deeper and faster.



ω-k SPACE

ω – k space is the frequency – wavenumber domain which is obtained by taking Fourier transform from time-space domain. In the ω – k space, we can analyze the previous study of surface wave dispersion relation in equation (1). A constant phase velocity can be represented by a straight line, $c = \omega / k$. The fundamental mode and higher modes can also be represented.



Higher frequency makes the mode approach c_1 asymptotically (i.e. no sensitivity).

Conversely, smaller frequency makes the mode approach c_2 asymptotically, which means low frequency waves are more sensitive to half space and evanescent waves at low frequency sample deep in the half space. We can consider three cases: fixed frequency, fixed wavenumber, and fixed wave speed. (1) At fixed frequency, waves propagate along different directions consistent with each discrete wavenumber. Discrete wavenumber represents discrete phase speed, $c_n = \omega / k_n$. The smaller wavenumber has the higher phase velocity. The lowest velocity corresponds to fundamental mode and high velocity comes from higher modes. (2) At fixed wavenumber, waves propagate along a fixed direction but with a discrete number of wave frequencies. The higher frequency has the higher phase velocity. The lowest velocity comes from fundamental mode and higher modes give higher velocities. (3) At fixed wave speed, there are infinite numbers of modes. For a given distance, waves of low frequency of the fundamental mode can arrive at the same time as higher modes at a higher wave frequency. Note that the phase velocity is described by a straight line.

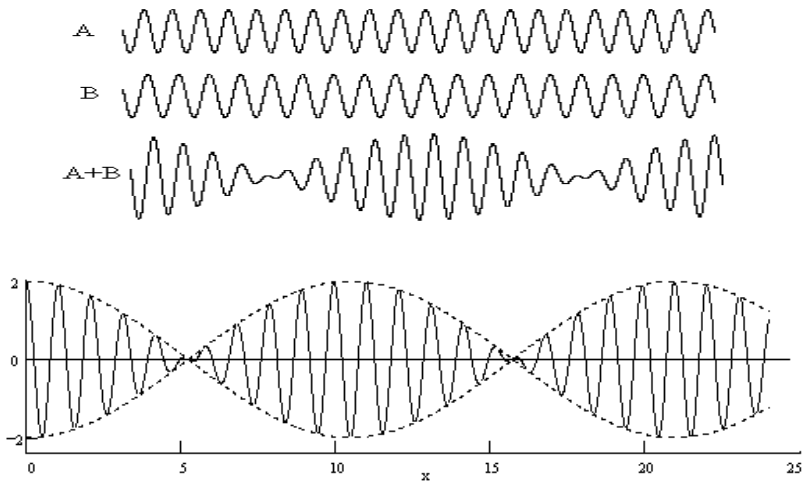
PHASE VELOCITY VS. GROUP VELOCITY

Consider two waves with particular wavenumber k_1 and k_2 and with particular wave frequency ω_1 and ω_2. The two waves can be represented by cosine term.

$$u(x,t) = \cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t) = 2\cos(kx - \omega t)\cos(\delta k x - \delta \omega t) \qquad (4)$$

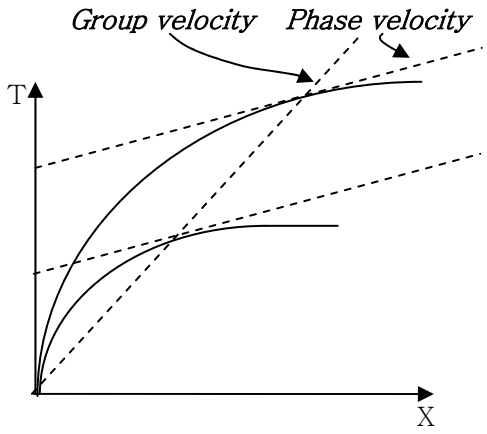
$$\delta\omega = \omega - \omega_1 = \omega - \omega_2, \omega_1 < \omega < \omega_2 \ , \ \delta k = k - k_1 = k - k_2, k_1 < k < k_2$$

The first cosine term travels with **phase velocity**, $c = \omega / k$ and the second travels with **group velocity**, $u = \delta\omega / \delta k$. The following figure describes the 'beating' effect.



The solid line has phase velocity and the dotted line has group velocity.

We can understand the phase and group velocity on the time-space plane and frequency-wavenumber plane, respectively.



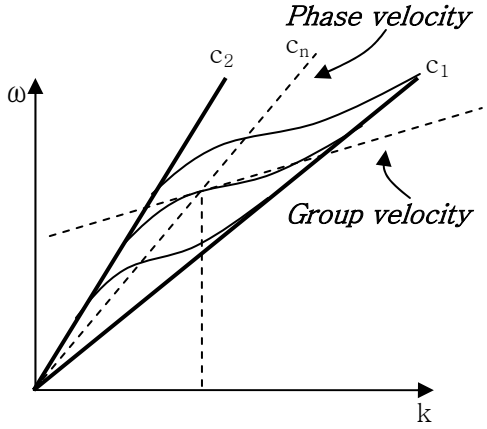
In time-space plane, the tangent dashed line represents the phase velocity.

$$\frac{dT}{dx} = p = \frac{1}{c} = \frac{k}{\omega} \tag{5}$$

The group velocity is defined by the following.

$$U = \frac{\delta\omega}{\delta k} = \frac{X}{T} \tag{6}$$

The phase velocity is faster than the group velocity.



In ω-k space, the group velocity is the tangent dashed line which is clear from equation (6). As frequency goes to infinity, group velocity is equal to phase velocity.

$$\lim_{\omega \to \infty} U = \lim_{\omega \to \infty} \frac{\delta\omega}{\delta k} = \frac{\omega}{k} \tag{7}$$

In this case, group velocity will be equal to c_1 for high frequency waves.