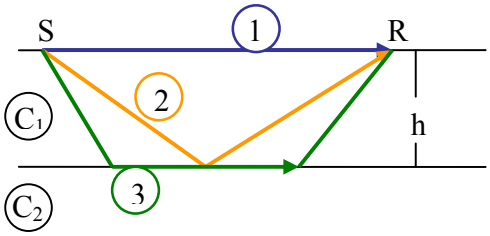
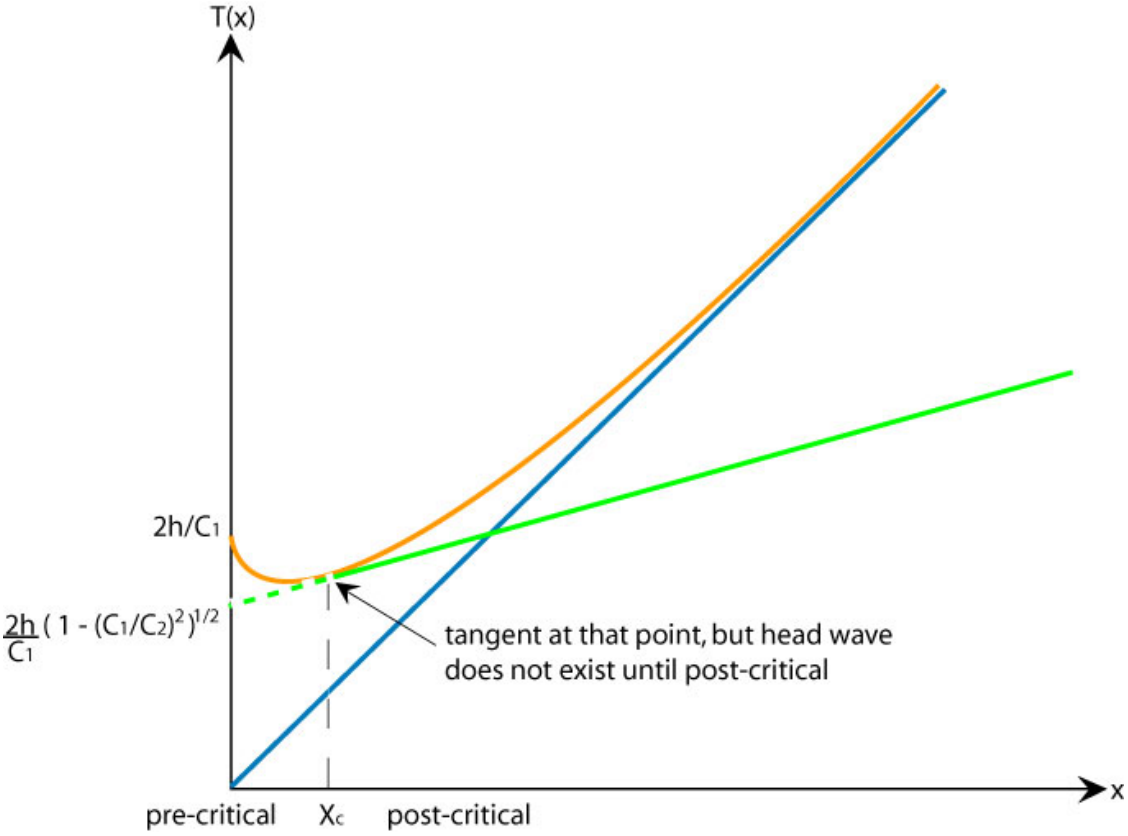
From last class:

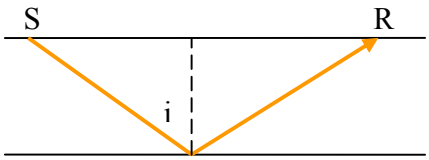


(1) Direct wave, simple function of time vs. distance
(2) Reflected wave, the asymptotic limit becomes like direct wave
(3) Head wave, straight line with slope $1/C_2$

Travel time curve:

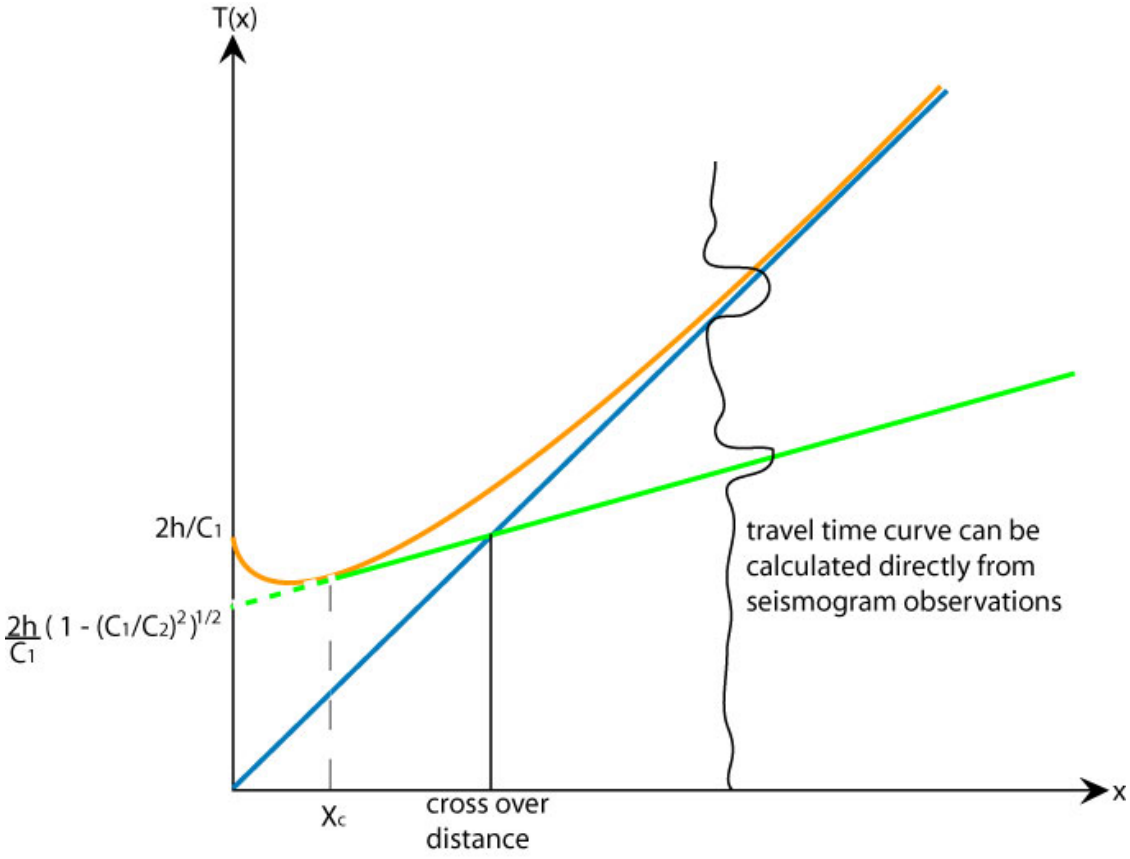


Critical angles:



(1) $i < i_c$
(2) $i = i_c$
(3) $i > i_c$

(1) If $i < i_c$, only have 2 arrivals, the direct wave and the reflected wave. Also, η_2 is a real number. There is no evanescence. For the reflected wave there is no phase shift, $\varepsilon = 0$.
(2) If $i = i_c$, begin to create the horizontal head wave as the angle becomes post-critical. The head wave still does not exist at this point or, in other words, at $i = i_c$, the head wave = the reflected wave.
(3) There is a critical distance X_c when $i = i_c$, after which you have a head wave and a reflected wave. Distances > X_c are post-critical, and $i > i_c$.



at $X=X_c$, the phase shift, $\varepsilon = 0$.

$\lim i_1 \Rightarrow \pi / 2$

$x \to \infty$

$\lim \varepsilon \Rightarrow g_0{}^0$

$x \to \infty$

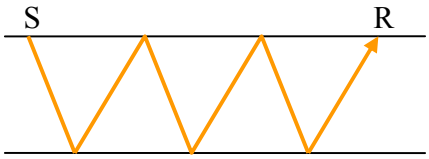
The cross-over distance is found by setting $T_{direct} = T_{head}$

$$x_c = 2h\left(\frac{c_2 + c_1}{c_2 - c_1}\right)^{1/2}$$

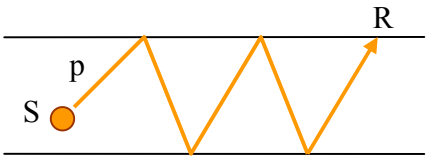
Thus we can solve the inverse problem of finding the velocity structure at depth from the observation of the seismogram. One can draw the travel time curve and, from the slope, calculate wave speeds lining up seismograms as a function of distance. Can calculate time intercept, knowing wave speeds, and get the layer thickness. Very quickly you can thus derive wave speed and layer thickness you need to know from the seismogram.

<u>Some issues that may arise:</u>

(1) Multiples

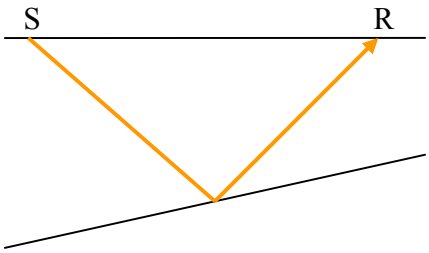


Your seismogram can have multiples. This is a big deal in exploration seismology. When there are complex structures it is not trivial to remove the multiple arrivals.

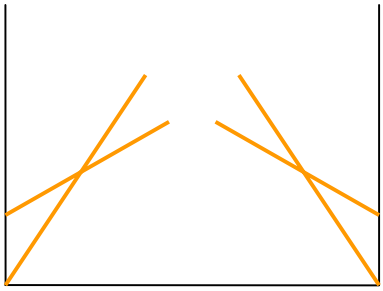


If you have a source at depth, you can have ghost multiples

(2) Dipping layer

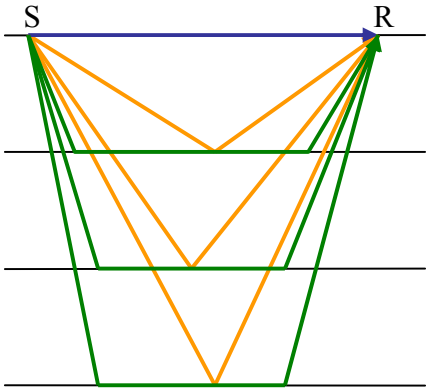


This is similar to parallel layers, but one layer is dipping. In the field, you can get around this by shooting the source twice, once on the right and once on the left, using the same geometry. This will allow you to determine C_1 and C_2 and get the slope of the layer.



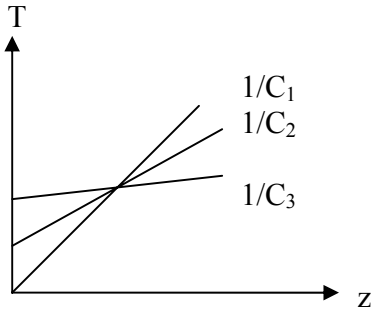
S	r_1	r_2	r_3	r_4
r_4	r_3	r_2	r_1	S

(3) Multiple Layers



At layer 1, you will see direct, reflected, and head waves.

At layers 2 and 3, you will get reflected and head waves



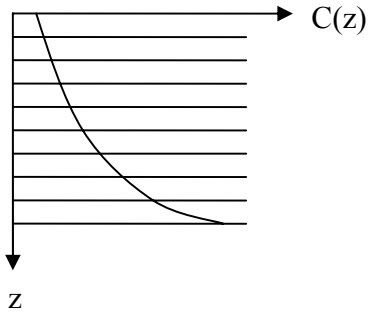
$$T_{head} = \frac{x}{C_2} + \frac{2h}{C_1}\left(1 - \left(\frac{C_1}{C_2}\right)^2\right)^{1/2} = px + 2h\eta$$

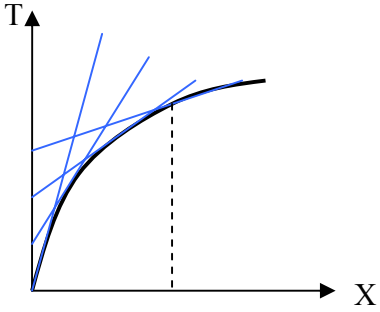
$$T = px + 2\sum_{i=1}^{n} h_i \eta_i$$

This is related to dix formula. You can use this to get layer thickness, from wave speeds. If the layers are very thin, it can become a continuum → wave speed increasing with depth.

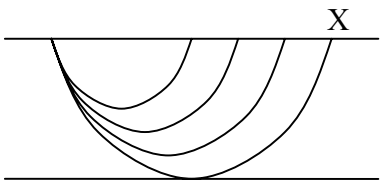
Other problems include:

"Blind layers". This happens when you have a very thin layer or a low velocity zone (LVZ).
Extension to continuous change in wave speed is now easy to solve.





Continuum, very thin layers with increasing wave speeds…



Bottom point, turning depth or bottoming depth

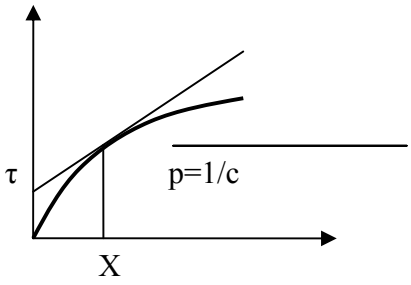
For layered system;
Tension → 1/c , i.e. related to slowness

$$1/c = p = \frac{\sin i}{c} = \frac{\sin i(r)}{c(r)}$$ at distance r

You can get information about where the wave goes horizontal at this point,

$$i_c = \frac{\pi}{2} \Rightarrow p = \frac{1}{c_{bottompoint}}$$

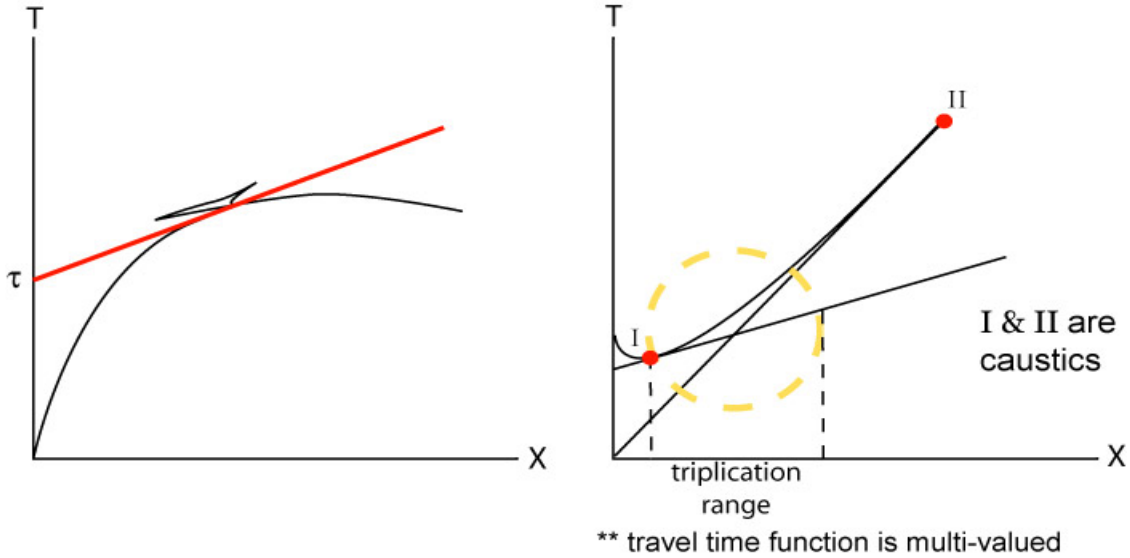
Can use this to get wave speed at a certain depth…get a c(z) profile by looking at several r's and bottom points. Forms principle of classical inversion technique to field c(z) from observed travel times.



$T(x) = \tau + px$
$= \tau + dT/dX\ x$

The intercept relates to τ… can be complicated as to whether there is a smooth increase in the wave speed or a discontinuous increase, or a LVZ.

bow tie- low velocity over high velocity& wave speed is increasing fast enough, get something that looks like this:

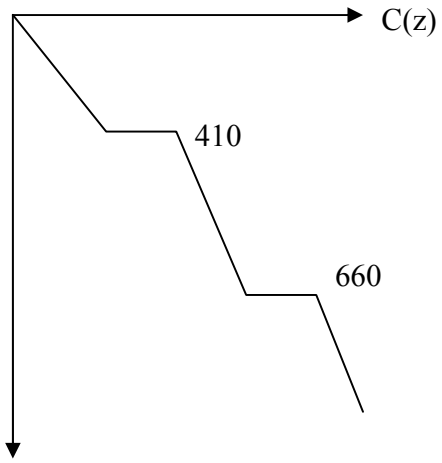


Triplication – 3 arrivals so that there is no unique correlation between travel time and distance, but unique correspondence between τ and p. Caustic at any particular distance where the energy from 3 waves comes together.

There is a technique in modern math that will allow you to deal with this non-uniqueness. As this only occurs in the space domain, 4D (x,t), you have "multi-pathing", or multiple paths to particular observed points. To get around this, you can go to a higher order by taking into consideration direction/slowness (p) and the phase shifts, to look at it in 6D space, (x,t,p,ω). This is referred to as PHASE SPACE – if you do all of your analyses in 6D space, the caustic does not appear. Each ray has a unique p and this will give a unique relationship between arrival and ray. Found in early 70's by mathematicians, aka microlocal analysis and there is a whole class of math that deals with this.

Seismologists just tend to avoid the distances where this is a problem, i.e. regional vs. teleseismic distances. The above technique would be useful for imaging the base of a salt dome (petroleum exploration) or even could image magma chambers.

Example:



Here are two triplications due to discontinuities. It can get really complicated. You cannot invert this data, BUT you can make synthetics and solve the forward problem and calculate what the wave form would look like.