TODAY'S LECTURE

1. Fault geometry
2. First motions
3. Stereographic fault plane representation
4. Moment tensor
5. Radiation patterns

FAULT GEOMETRY

The fault geometry is described in terms of the orientation of the fault plane and the direction of slip along the plane. The geometry of this model is shown in Figure 1.

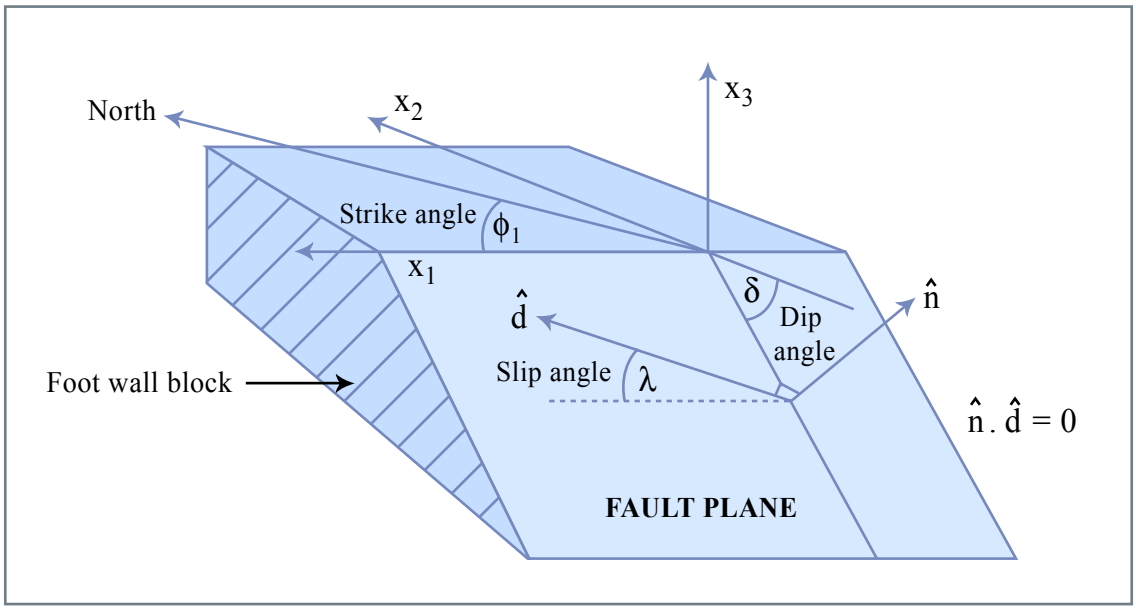


Figure by MIT OCW.

Figure 1. Fault geometry used in earthquake studies. [Adapted from Stein and Wysession, 2003]

$\hat{n}$ is *normal vector* of the fault plane. $\hat{d}$ is *slip vector* which indicates the direction of motion of hanging wall block. x_1 axis is in the fault strike so ϕ is *strike angle*. The *dip angle* δ gives the orientation of the fault plane with respect to the surface. The *slip angle* λ gives the motion of the hanging wall block with respect to the foot wall block. The motion is called *left-lateral* for $\lambda = 0$, *right-lateral* for $\lambda = 180$, *normal faulting* for $\lambda = 270$, and *reverse* or *thrust faulting* for $\lambda = 90$. Most earthquakes consist of some combination of these motions and have slip angles between these values. Note that the basic fault types can be related to the orientations of the principal stress directions. Actual fault geometries can be much more complicated. Such complicated seismic events can be treated as a superposition of the simple events.

FIRST MOTIONS

The focal mechanism uses the fact that the pattern of radiated seismic waves depends on the fault geometry. The simplest method is the first motion, or polarity, of body waves. Figure 2 illustrates the first motion concept for a strike-slip earthquake on a vertical fault.

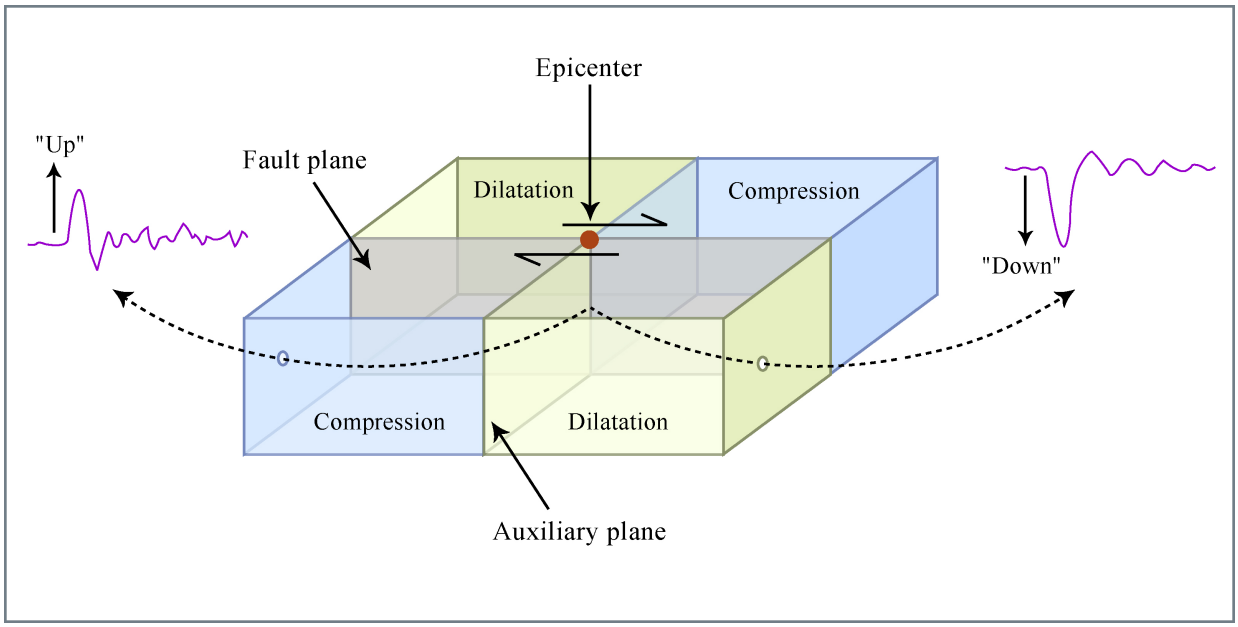


Figure by MIT OCW.

Figure 2. The relation between the first motion and fault geometry [Adapted from Stein and Wysession, 2003]

The first motion is *compression* when the fault moves "toward" the station and *dilatation* for "away from". A vertical seismogram records are upward for compression and downward for dilatation. A problem is that the first motion on actual fault plane is the same as that on the *auxiliary plane* which is perpendicular to fault plane, so the first motions alone cannot resolve which plane is the actual fault plane. This is a fundamental ambiguity in inverting seismic observations for fault models. We need additional geologic or geodetic information such as the trend of a known fault or observations of ground motion.

STEREOGRAPHIC FAULT PLANE REPRESENTATION

The fault geometry can be found from the distribution of data on a sphere around the focus. A stereographic projection transforms a hemisphere to a plane. The graphic construction is a stereonet (Figure 3).

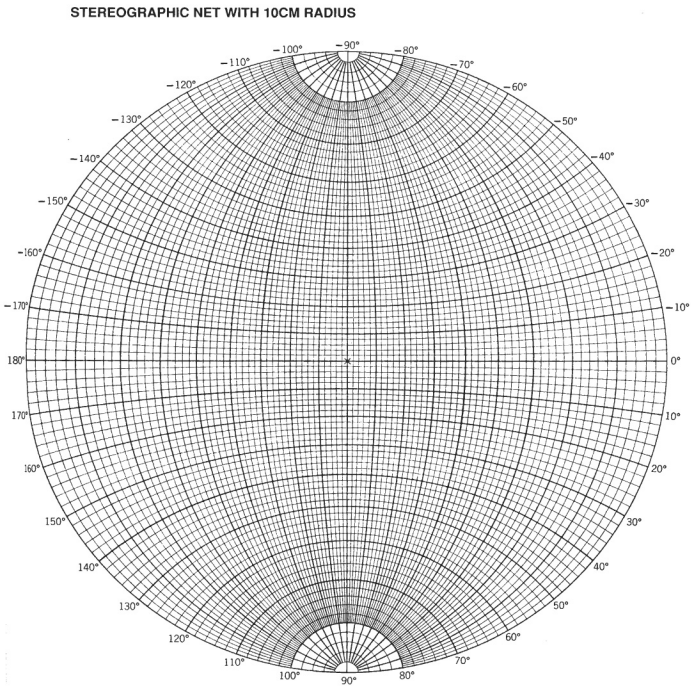


Figure 3. A stereonet used to display a hemisphere on a flat surface

Using the stereonet, we can obtain the projection. Figure 4 illustrates the stereographic projection.

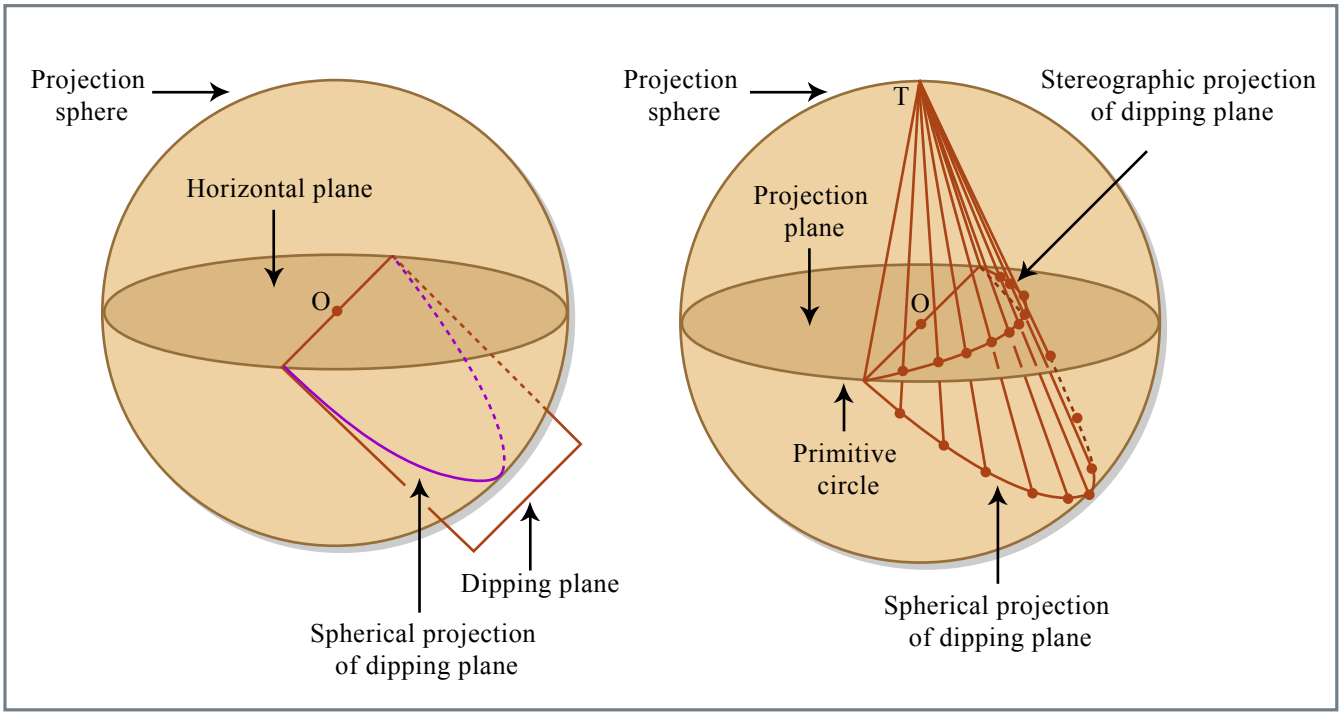


Figure by MIT OCW.

Figure 4. The stereographic projection.
[http://hercules.geology.uiuc.edu/~hsui/classes/geo350/lectures/earthquakes/earthquakes.html]

We can also plot planes perpendicular to a given plane by rotating the stereonet and finding the point on the equator 90° from the intersection of the plane with the equator. Any plane through this point can be perpendicular to the plane. Thus, we can get the focal mechanisms for earthquakes with various fault geometries. Figure 5-(a) shows the three ideal focal mechanisms. Various focal mechanisms can be possible according to various geometry.

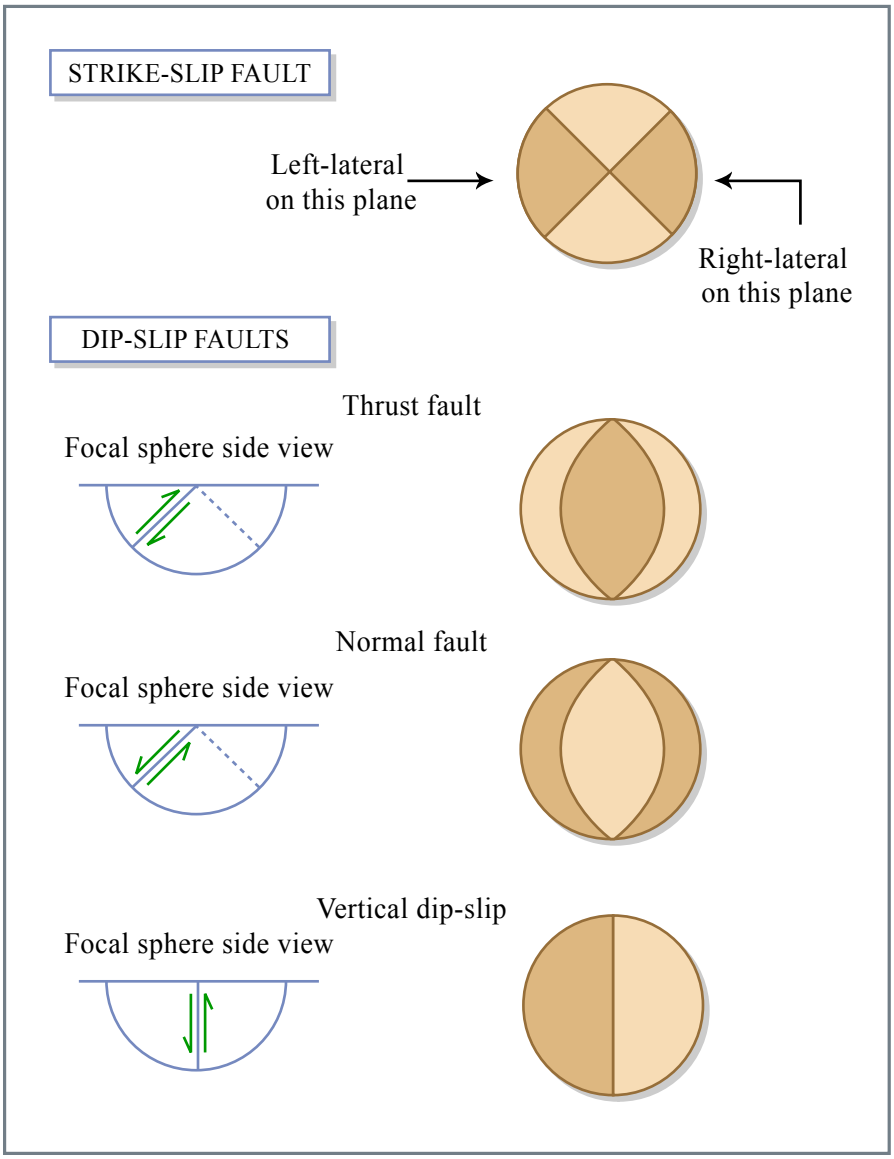


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(a) (b)

Figure 5. (a) The stereonet of different types of faults. (b) Focal mechanisms and some seismograms for three different earthquakes. Compressional quadrants are shown shaded. [Adapted from Stein and Wysession, 2003]

In reality, we plot the points where rays intersect the focal sphere, so that the nodal planes can be found, considering the ray as compression (upward first motion) or dilatation (downward first motion). Figure 5-(b) illustrates the focal mechanisms and seismograms for three different earthquakes.

MOMENT TENSOR

To know the source properties from the observed seismic displacements, the solution of equation of motion can be separated as below

$$u_i(\vec{x},t) = G_{ij}(\vec{x},t;\vec{x}_0,t_0) f_j(\vec{x}_0,t_0) \quad (1)$$

u_i is the displacement, f_j is the force vector. The Green's function G_{ij} gives the displacement at point x that results from the unit force function applied at point x_0. Internal forces f must act in opposing directions, $-f$, at a distance d so as to conserve momentum (*force couple*). For angular momentum conservation, there also exists a complementary couple that balances the forces (*double couple*). There are nine different force couples as shown in Figure 6.

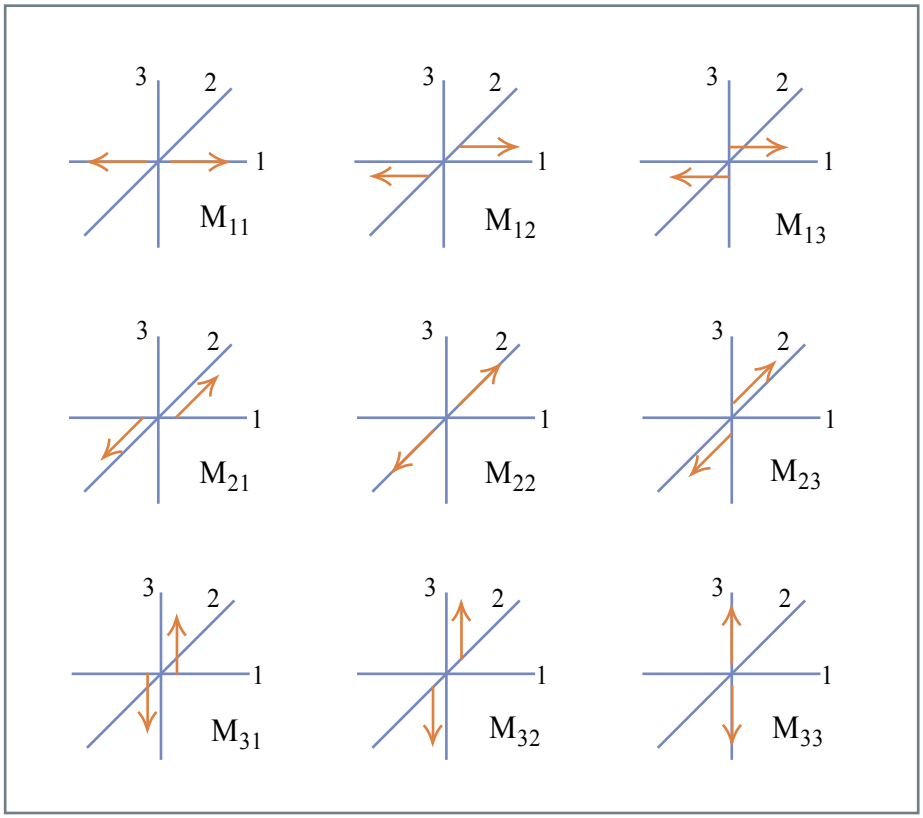


Figure by MIT OCW.

Figure 6. The nine different force couples for the components of the moment tensor. [Adapted from Shearer, 1999]

We define the *moment tensor* M as

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \quad (2)$$

M_{ij} represents a pair of opposing forces pointing in the i direction, separated in the j direction. Its magnitude is the product fd [unit: Nm] which is called *seismic moment*.

For angular momentum conservation, the condition $M_{ij} = M_{ji}$ should be satisfied, so the momentum tensor is symmetric. Therefore we have only six independent elements. This moment tensor represents the internally generated forces that can act at a point in an elastic medium.

The displacement for a force couple with a distance d in the $\hat{x}_k$ direction is given by

$$u_i(\vec{x},t) = G_{ij}(\vec{x},t;\vec{x}_0,t_0) f_j(\vec{x}_0,t_0) - G_{ij}(\vec{x},t;\vec{x}_0 - \hat{x}_k d,t_0) f_j(\vec{x}_0,t_0) = \frac{\partial G_{ij}(\vec{x},t;\vec{x}_0,t_0)}{\partial x_k} f_j(\vec{x}_0,t_0) d \quad (3)$$

The last term can be replaced by moment tensor and thus

$$u_i(\vec{x},t) = \frac{\partial G_{ij}(\vec{x},t;\vec{x}_0,t_0)}{\partial x_k} M_{jk}(\vec{x}_0,t_0) \quad (4)$$

There is a linear relationship between the displacement and the components of the moment tensor that involves the spatial derivatives of the Green's functions. We can see the internal force f is proportional to the spatial derivative of moment tensor when compared equation (1) with (4).

$$f_i \sim \frac{\partial}{\partial x_j} M_{ij} \quad (5)$$

Consider right-lateral movement on a vertical fault oriented in the x_1 direction and the corresponding moment tensor is given by

$$M = \begin{bmatrix} 0 & M_{12} & 0 \\ M_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

where $M_0 = \mu \overline{D} s$ called *scalar seismic moment* which is the best measure of earthquake size and energy release, μ is shear modulus, $\overline{D} = D(x)/L$ is average displacement, and s is area of the fault. M_0 can be time dependent, so $M_0 = \mu \overline{D}(t) s(t)$. The right-hand side time dependent terms become *source time function*, $x(t)$, thus the *seismic moment function* is given by

$$M(t) = M_0 x(t) = \mu D(t) s(t) \quad (7)$$

We can diagonalize the moment matrix (6) to find principal axes. In this case, the principal axes are at 45° to the original x_1 and x_2 axes.

$$M' = \begin{bmatrix} M_0 & 0 & 0 \\ 0 & -M_0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

Principal axes become tension and pressure axis. The above matrix represents that x_1' coordinate is the tension axis, T, and x_2' is the pressure axis, P. (Figure 7)

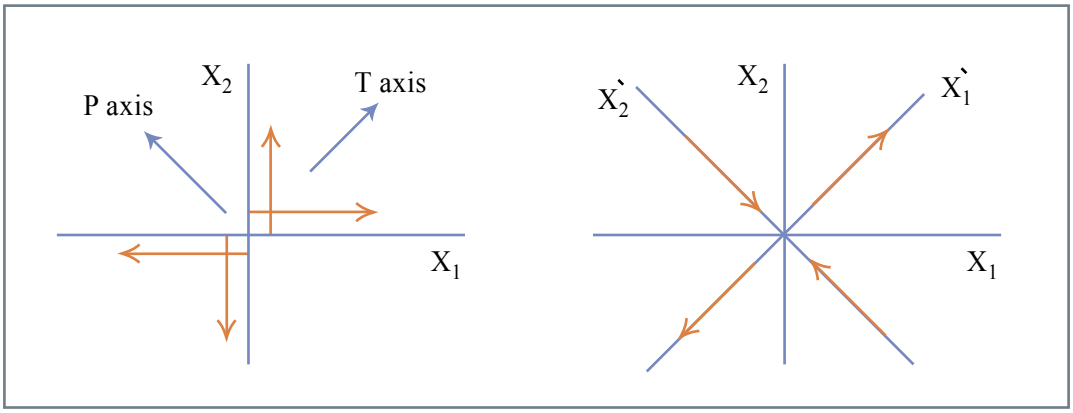


Figure by MIT OCW.

Figure 7. The double-coupled forces and their rotation along the principal axes. [Adapted from Shearer, 1999]

RADIATION PATTERNS

P-wave potential in spherical coordinate is given by

$$\phi(r,t) = \frac{-f(t-r/\alpha)}{r} = \frac{-f(\tau)}{r} \tag{9}$$

where α is the P wave velocity, r is the distance from the point source, and τ is time residual. Therefore, the displacement field is given by the gradient of the displacement potential $u = \partial\phi/\partial r$

$$u(r,t) = \left(\frac{1}{r^2}\right) f(\tau) + \left(\frac{1}{r\alpha}\right) \frac{\partial f(\tau)}{\partial \tau} \tag{10}$$

The first term in the right hand side is near field displacement because of the decay as $1/r^2$ and the last term is far field displacement with the decay as $1/r$. When we consider the relation between internal force and moment tensor given by equation (5), we can find that the near field term has no time dependence but the far field term has time dependence. The relations are given by

$$f \sim \frac{\partial M}{\partial x} \quad , \quad \frac{\partial f}{\partial \tau} \sim \frac{\partial M}{\partial \tau} \sim \dot{M}(t) \tag{11}$$

Therefore, the near field term represents the permanent static displacement due to the source and the far field term represents the dynamic response or transient seismic waves that are radiated by the source that cause no permanent displacement. Figure 7

represents the near and far field behaviors.

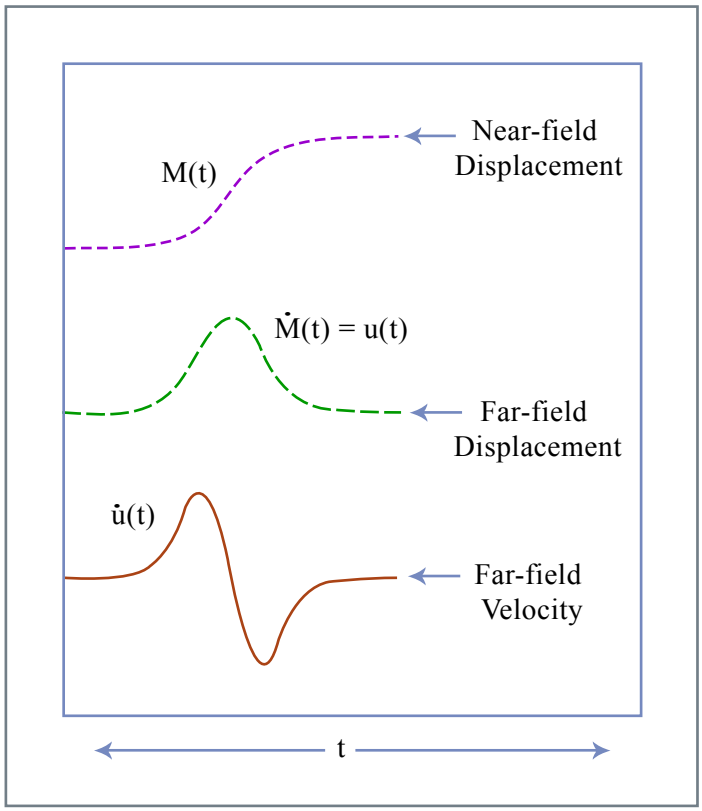


Figure by MIT OCW.

Figure 7. The relationships between near-field and far-field displacement and velocity. [Adapted from Shearer, 1999]

In spherical coordinates, far field displacement is given by

$$u_r = \frac{1}{4\pi\rho\alpha^3 r}\dot{M}(t - r/\alpha)\sin 2\theta\cos\phi \qquad (12)$$

The first amplitude term decays as $1/r$. The second term reflects the pulse radiated from the fault, $\dot{M}(t)$, which propagates away with the P-wave speed α and arrives at a distance r at time $t - r/\alpha$. $\dot{M}(t)$ is called the *seismic moment rate function* or *source time function*. Its integration form in terms of time is given by equation (7). The final term describes the P-wave radiation pattern depending on the two angle (θ, ϕ). At $\theta = \phi = 90^\circ$, the displacement is zero on the two nodal planes. The maximum amplitudes are between the two nodal planes. Figure 8 represents the far-field radiation pattern for P-waves and S-waves for a double-couple source.

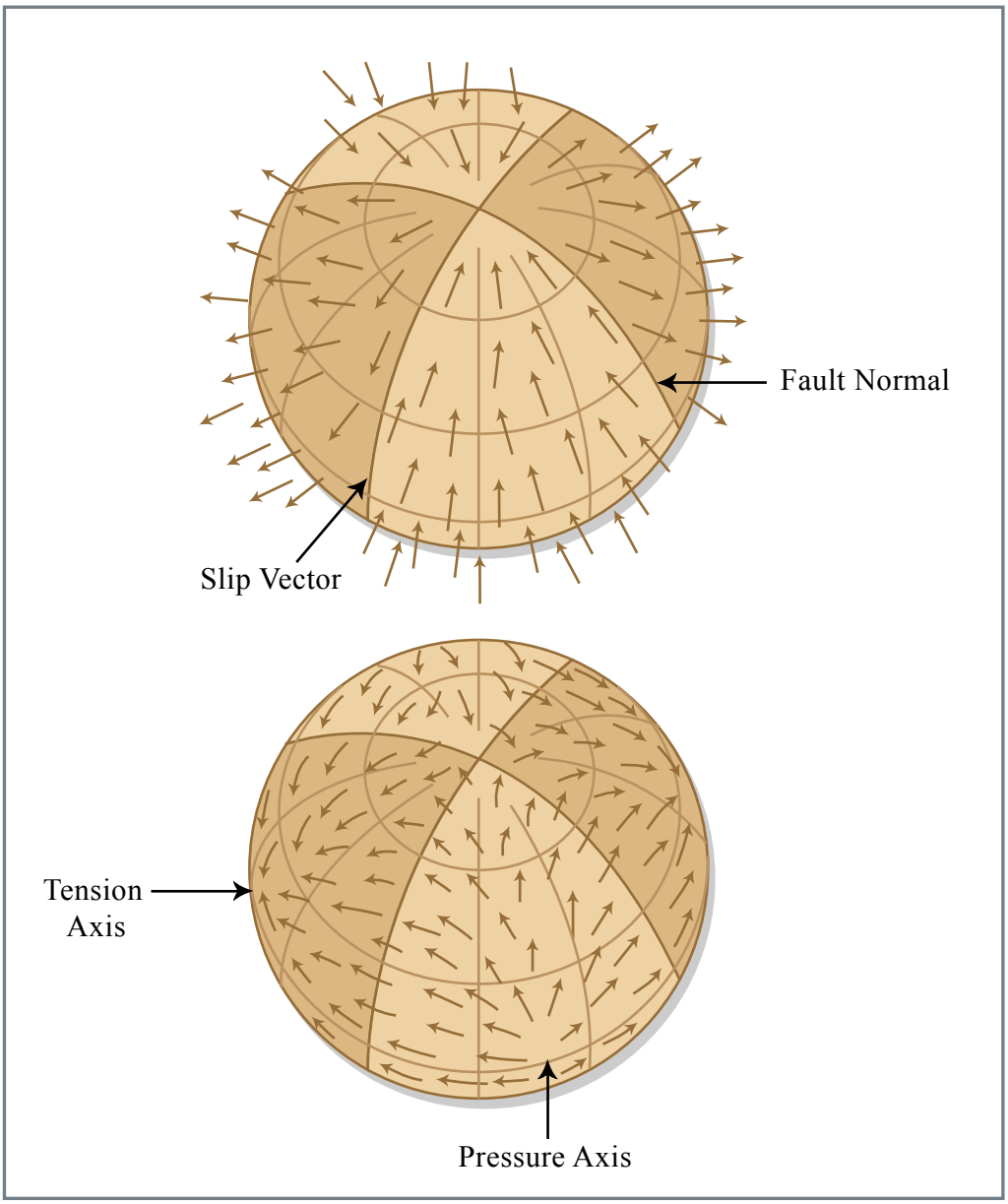


Figure by MIT OCW.

Figure 8. The far-field radiation pattern for P-waves (top) and S-waves (bottom) for a double-couple source. [Adapted from Shearer, 1999]

Reference

Stein S. and M. Wysession, *An introduction to seismology, earthquakes, and earth structure*, Blackwell Publishing, 2003.

Shearer P.M., *Introduction to seismology*, Cambridge University Press, 1999.