Earthquakes

This lecture will cover earthquakes – what they are and how they are used – and next lecture will cover how they tie into wave propagation.

I. Societal Issues

Detailed study of earthquakes gives rise to more information about societal issues (such as seismic hazard or risk), tectonics (in the broad sense, e.g. stress fields, as well as on a smaller scale such as in hydrocarbon reservoirs and geothermal energy plants), and the wave motion generated by the earthquake.

- The "beach ball" diagrams of focal mechanisms can give information about the type and orientation of earthquakes such as the tsunami-triggering earthquake suffered by Sumatra.
- In the San Francisco Bay Area, there are several kilometers of separation between the San Andreas Fault and the Hayward Fault, creating a wide rupture zone.
- The East African Rift System creates extensional stress in East Africa as well as Atlantic and South Atlantic spreading. This system also causes motion of the Indian plate. Also note that extension implies that the African plate must grow, so the plate geometries are changing.

II. Earthquake Location and Focal Mechanisms

Once seismic information is recorded, two questions must be answered.

1. How do you locate earthquakes?
2. How do you reconstruct focal mechanisms?

A. Tectonic Frame

First, let's get back into the tectonic frame. The equation of motion (Cauchy or Navier) is given by

$$\rho \frac{\delta^2 u_i}{\delta t^2} = \delta_j \tau_{ij} + f_i \tag{1}$$

where f_i represents the body forces such as gravity and electromagnetism due to coupling between the seismic and electric fields. Setting the body forces to zero leads to

$$\rho \frac{\delta^2 u_i}{\delta t^2} - \delta_j \tau_{ij} = 0 \tag{2}$$

In theory, we try to find the equivalent body forces that best describe the fault motion and put them into equation (1). This is formally known as the "Representation Theorem" which is further discussed in, e.g., Ahi and Richards.

B. Green's Function

For a point source at $(x',\, t')$, the solution for the equation of motion is given by a *Green's Function* $G(x,t)$. The force can be represented as follows:

$$f_i = A\delta(t-t')\delta(x-x')\delta_{in} \tag{3}$$

where A is the amplitude, $(t-t')$ is time, $(x-x')$ is position, and n is the direction. Putting this into the equation of motion and solving for u_i (the displacement field resulting from wave motion due to a point source) gives the Green's Function:

$$u_i(x,t) = u_i \equiv G_{ij}(x,t\mid_{x',t'}) \tag{4}$$

A seismogram can be built by integrating over the Green's Functions. Now, consider a source at $(x',\, t')$ giving a displacement field $u(x,t)$. We get the equation

$$u_i(x,t) = G_{ij}(x,t\mid_{x',t'})f_i(x',t') \tag{5}$$

Equation (5) is an important relationship because it summarizes how the source can be used. For example, if we know the source, we can calculate displacement using forward modeling (i.e. calculating a synthetic seismogram). If the interest is the source and not the wave propagation, and we know the Green's Functions, we can use the displacement to get information about the seismic source (i.e. inversion for seismic source parameters). A full inversion involves the inversion of the Green's Functions (the waveforms). For details see Stein and Wysession, Chapter 4.

Now, let's answer the questions of earthquake locations and focal mechanisms using classical seismology. We are looking for the locations of the epicenter (in terms of latitude θ and longitude γ) and the hypocenter m (in terms of θ, γ, time t_0, and depth h). Location can be found using the travel time difference $T_p - T_s$. We can choose the arrival times of S and P waves and compare them to distances on travel time curves, i.e. take the distance $T_p - T_s$ from the seismogram and find the corresponding Δ value on the plot of the travel time curve. The epicenter is calculated using Δ .

C. Depth Phases

Earthquake location can also be found using *depth phases*. A depth phase is a seismic phase that only occurs if the earthquake has a certain depth. pP, sP, sS, pS, etc. have depth phases related to P and S.

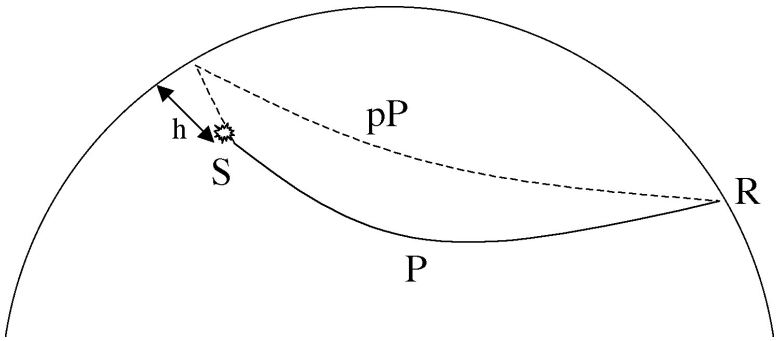


Figure 1

Figure 1 shows paths of both a P wave and a pP wave for $h \neq 0$. At very large surface distance, the two curves travel on nearly the same path, so $T_{pP} - T_p = \delta t \sim \frac{2h}{v}$. Note that this corresponds with the travel time curves discussed previously:

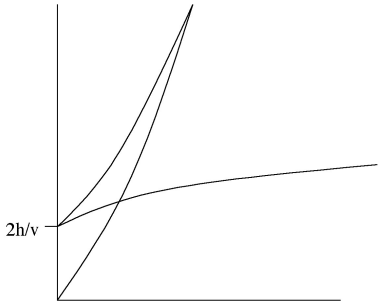


Figure 2

Problems arise when the source is too shallow, giving a low h-value. When this happens, the travel time difference is small and makes it difficult to distinguish between the two arrivals. Using a combination of depth phases and travel time differences can give a good estimate of the hypocenter of the earthquake.

D. Refinement of Source Location

The hypocenter m_0 is described by $m_0 = (t_0, h_0, \theta_0, \gamma_0)$, so

$$T(m_0) + dT = T(m) \tag{6}$$

If we have a travel time T related to the best guess m_0 of the location m, we get

$$dT = \frac{\delta T}{\delta t_0} dt_0 + \frac{\delta T}{\delta h} dh + \frac{\delta T}{\delta \theta} d\theta + \frac{\delta T}{\delta \gamma} d\gamma \tag{7}$$

or,

$$dT = (\bigtriangledown' T) \cdot \mathbf{dm} \tag{8}$$

where $\mathbf{dm} = (dt_0, dh, d\theta, d\gamma)$. $d = Am$, where A is the sensitivity matrix, gives

$$\hat{m} = (A^T A)^{-1} A^T d \tag{9}$$

Because the dependence of the travel time on the origin time is 1,

$$\frac{\delta T}{\delta t} = 1 \tag{10}$$

To find the other values, substitutions can be made as follows:

$$\frac{\delta T}{\delta h} = \frac{\sqrt{\frac{r^2}{v^2} - p^2}}{r} = \frac{1}{r}\sqrt{\eta^2 - p^2} \tag{11}$$

$$\frac{\delta T}{\delta \theta} = \frac{-r \sin i}{v} \cos z = -p \cos z \tag{12}$$

$$\frac{\delta T}{\delta \gamma} = \frac{-r \sin i}{v} \sin z \sin \theta = p \sin z \sin \theta \tag{13}$$

where z is the azimuth, $p = \frac{r \sin i}{v}$, $\eta = \sqrt{(\frac{1}{v})^2 - p}$, and $m_0 + dm = m$

The calculation of seismic sources has led to the following discoveries:

- Subduction zones exist, thus plate tectonic theory developed.
- There are no earthquakes deeper than 700km, leading to layered convection theory.
- Most earthquakes are shallow, and everything deeper than 100km must be part of a subduction zone.
- In subduction zones, there is high seismicity in shallow and deep regimes, but low seismicity in the intermediate region. This can be explained using the stress field of the plate, shown in Figure 3. For less than 410km depth, the subducting plate undergoes extension. Greater than 660km depth, the plate is compressed. The surrounding mantle from 0-410km is primarily olivine, from 410-660km is spinel and greater than 660km is perovskite.

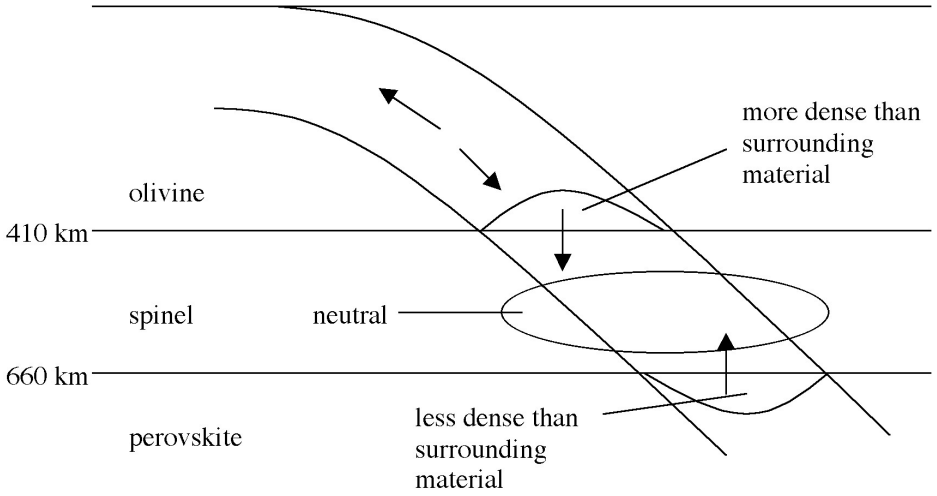


Figure 3

The cold subducting slab has a difference in density from the surrounding material at the phase change boundary depths, giving rise to the positive and negative buoyancy at the edges of the intermediate zone and creating a neutral buoyancy in between. Figure 4 illustrates the density difference using geotherms.

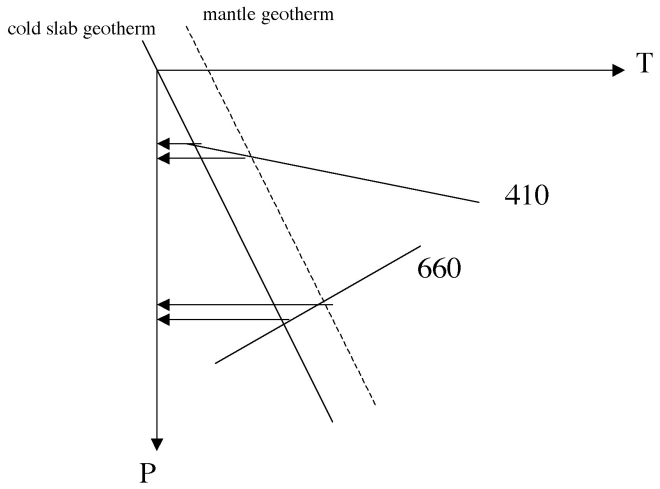


Figure 4

III. Elastic Rebound

In modern language, elastic rebound is referred to as "seismic cycles," moving through interseismic, preseismic, coseismic, and postseismic stages. Recent research uses GPS and InSAR, measuring motion at the fault zone as well as away from the fault.

Calculation of the primary stress values gives eigenvalues for each earthquake. The relative magnitudes of the earthquakes tells us the type of faulting. If the largest eigenvalue is vertical and the smallest is horizontal, we have normal faulting. If the smallest is vertical and the largest is horizontal, we get reverse faulting. If the largest and smallest are both horizontal, we have pure strike-slip faulting.

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