Introduction to Seismology: Lecture Notes
1st April 2005

CONTINUATION OF TOMOGRAPHY
Futher Comments:

1. Linearization of the travel time residual

$$\delta t = \int_{path} \Delta s \, dl$$

If we assume a simple reference model we can calculate the reference ray using Eikonal equations.

Figure 1

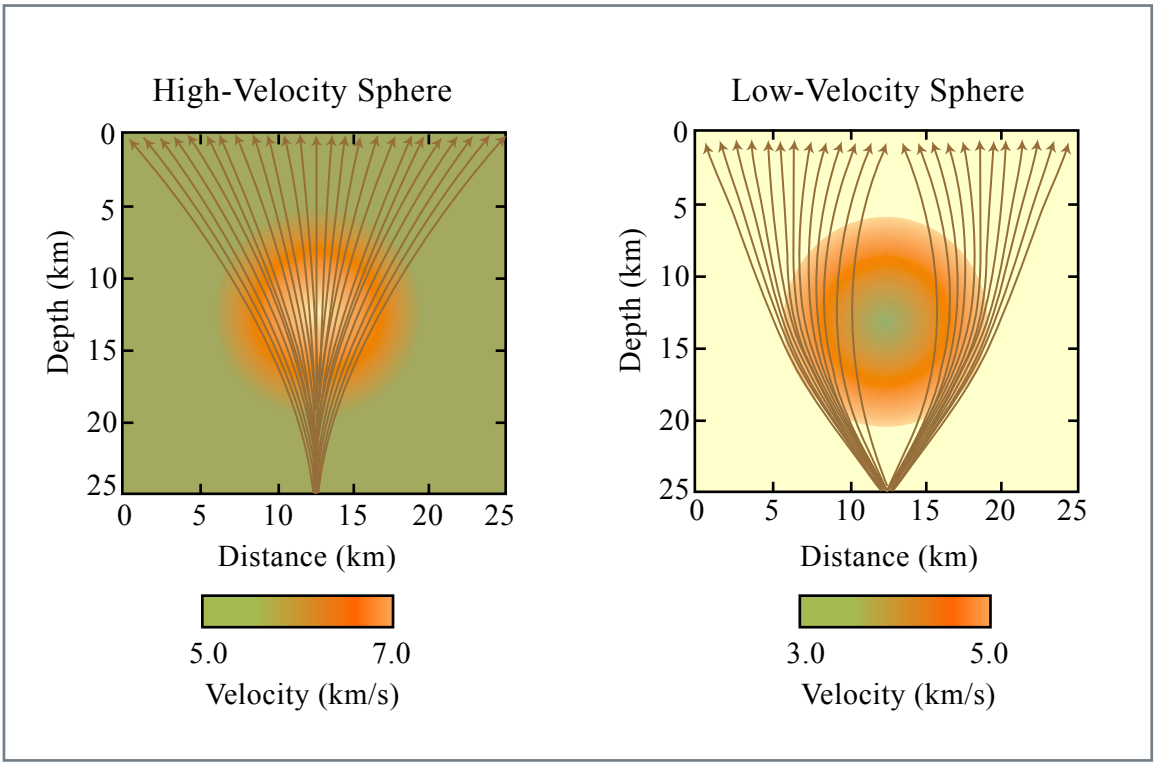


Figure by MIT OCW.

Structures in the subsurface will affect the propagation path of a ray. Figure 1a. shows the focusing effect of a high-velocity sphere on a bundle of rays which results in a high sampling rate for the body. An example of this effect in the Earth is the high velocity of cold, subducted slabs. Figure 1b. shows the opposite situation, a low-velocity sphere. The bundle of rays tends to avoid the structure and the sphere is less well sampled. An Earth example of this is a hot, low velocity mantle plume. The bending of the rays away from a low velocity body makes it difficult to image such features.

Follow the process:

$\hat{m} = \Delta\hat{s}$

a. Slowness, s, is known
b. Recompute Eikonal equations

c. Update the ray geometry
d. Repeat the exercise

$\delta t = T_{obs} - T_{3Dref}$ so the travel time residual becomes $\delta t \cong \int_{3Drefpath} \Delta s dl$ where $\Delta s = s - s_{3Dref}$

This is normally done once or twice for global, long wavelength tomography and multiple times for detailed tomography.

2. There are different strategies for Global Tomography. Each has advantages and disadvantages

a. **Global** (e.g. spherical harmonics) vs. **Local** (e.g. blocks, splines) parameterizaition.
   Global: (+) nice theory, good for long wavelengths, facilitates spectral analysis.
   (-) artifacts in poorly sampled regions, not suitable for high definition imaging of, say, slabs.
   Local: (+) deals better with uneven data coverage, can be regionalized to obtain high definition images in densely sampled regions.
   (-) poor constraints on long wavelengths, many unknowns.
b. **Direct** vs. **Interative** inversion algorithms
   Direct: (+) elegant resolution anaylsis, (-) RAM limits solution
   Interative: (+) "large" models, (-) indirect resolution analysis, subjection convergence criteria and regularization.
c. **Data**: Waveforms vs. routinely processed bulletin data.

Figure 2

Image removed due to copyright considerations.

Please see:
Kárason, H., and R. D. Van der Hilst. "Constraints on Mantle Convection from Seismic Tomography." In *The History and Dynamics of Global Plate Motion* (*Geophysical Monograph)*. Vol. 121. Edited by M. R. Richards, R. Gordon, and R. D. Van der Hilst. Washington, DC: American Geophysical Union, 2000, pp. 277-288.

Figure 2 illustrates the importance of cell size when approaching the question you are trying to answer. Decreasing your cell size will result in more detailed images but there is the hazard of over-parameterization of the model if the cell size is too small. You want

the Resolution matrix, R, is approach the Identity matrix, therefore tailor your cell size towards these values.

Figure 3

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Figure 3 is an example of the use of different waveforms for tomography. The images are shown at a depth of 1300km. Fig. 3a & b. were calculated using S waves and Fig 3c. using P waves. Fig 3b & 3c. have similar parameters and show very similar results. Areas of fast velocities appear to correlate well with locations of long term subduction on Earth. At depths >1700km the two models show very different results. This is expected because of the different effect of temperature and composition on P and S waves.

Early tomography used only direct waves. Improvements can be made by combining many waveforms. For example pP, PP (good for sampling remote oceanic areas), Pdiff and PKP phases.

Surface Waves

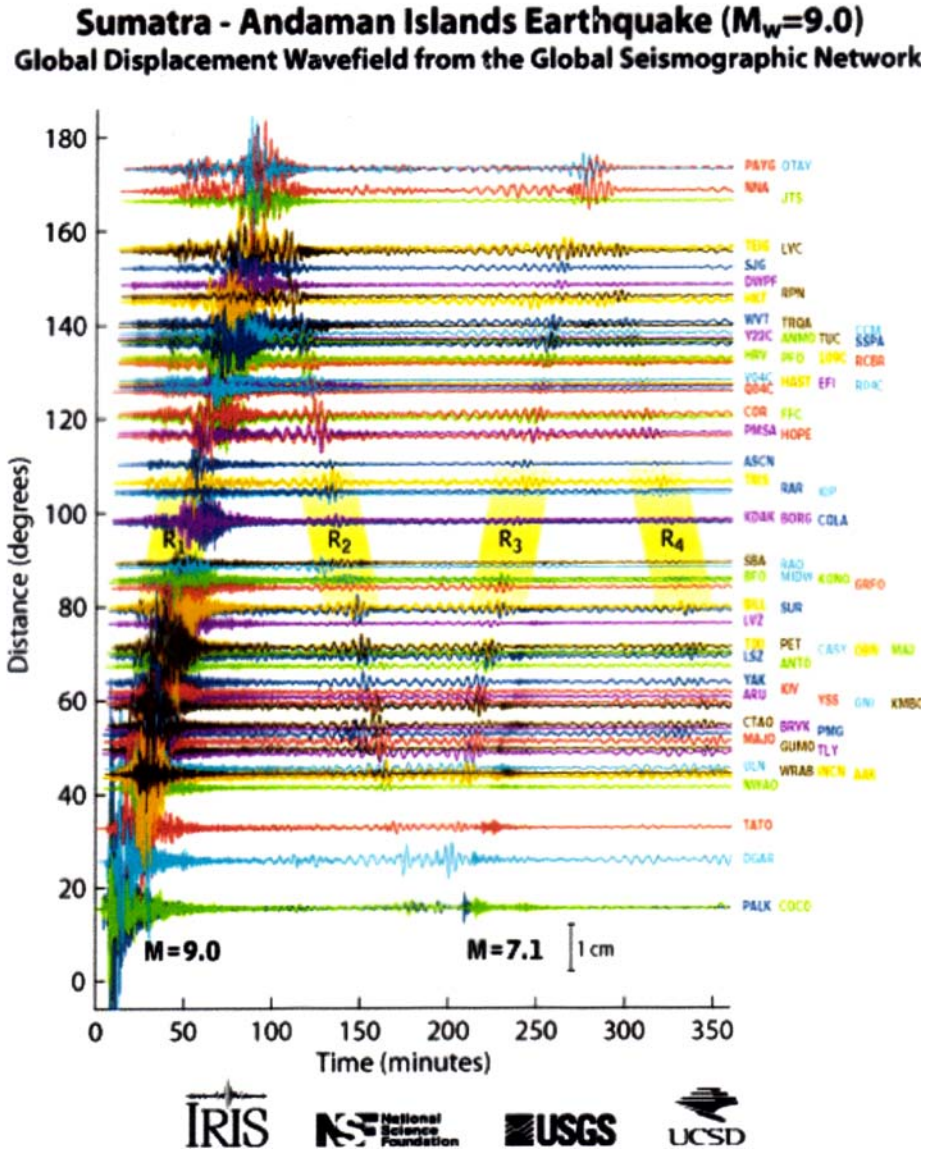
Figure 4

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Figure 4. shows the vertical component waveforms for an event recorded by an array in south-east China. The first arrival is the P waves followed by the S wave. The surface waves are clearly seen as the later, high amplitude arrivals.

Image removed due to copyright considerations.

Figure 5. 12/26/04 Sumatra earthquake location.

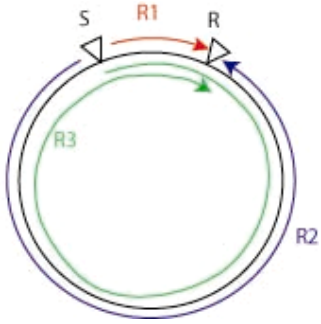


(from http://www.iris.iris.edu/sumatra/)

Figure 6. Low frequency, surface wave arrivals for the Sumatra earthquake. The surface waves are known as "ground roll" in the exploration industry.

Rn labeling refers to multiple global propagation paths. See Figure 7 for an explanation..

Figure 7



Surface waves decay in a 2D sense compared with body waves, which decay in a 3D sense.Therefore surface waves are able to maintain there amplitudes further than body waves. The period of surface waves used in tomography ranges from 30-200 secs, compared with only 1-5 secs for body waves.

Recall the effect of evanescence in post-critical situations. The vertical slowness becomes complex, $\eta = i\hat{\eta}$, and $\phi = \exp\{-\eta\omega z\}\exp\{i\omega(px - t)\}$. The frequency and depth dependence of φ result in high frequency waves decaying quickly with depth. Conversely low frequency waves decay more slowly and sample greater depths.

Figure 8

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In a T:x plot (Figure 8) the surface waves plot along the straight line (wide blue line) and the body waves plot as curves lines beneath the straight line.

The three-component data for an event located in Tonga is shown in Figure 9.

Figure 9

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The vertical component (Z) shows more clearly the arrival of the compressional waves (P, PcP, PP, PPP) and coupled P+Sv waves (ScP, Rayleigh waves(LR)). The first horizontal component (N) has not recorded many P wave arrivals because this receiver was located to the west of the event, resulting in a natural polarization. This seismogram is more rich in S wave arrivals (S, ScS, SS, SSS) and Love surface waves (LQ) from SH waves interacting with the free surface. This component is equivalent to the transverse component. The second horizontal component (E) is equivalent to the radial component Notice that the Love waves arrive early than the Rayleigh waves. More on this later in the class.

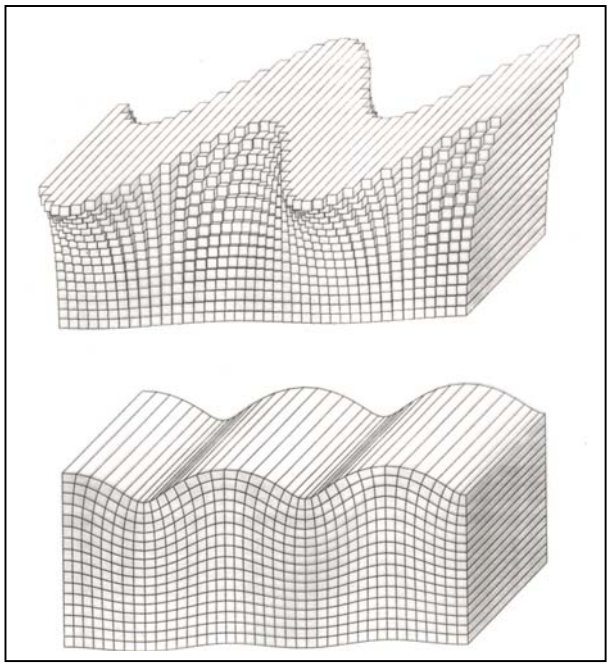


Figure 10

Love waves have a sideways motion (similar to a snake). Evanescence occurs and the amplitude decays exponentially with depth.

Rayleigh waves are analogous to ocean waves. They have retrograde or prograde motion.

LOVE WAVES

Love waves behave in the same way as ground roll or acoustic waves.

In an elastic medium $V_p = \sqrt{\dfrac{\mathrm{K} + \frac{4}{3}\mu}{\rho}}$ and the bulk sound speed is $V_{BSS} = \sqrt{V_p^2 - \frac{4}{3}V_S^2}$

In a fluid a P-wave becomes an acoustic wave. $V_p = \sqrt{\dfrac{\mathrm{K}}{\rho}}$ which is also the bulk sound speed, V_{BSS}. Examples of acoustic waves are core waves and T-phases.

ACOUSTIC WAVES

For an S wave $V_s = \sqrt{\dfrac{\mu}{\rho}}$

$$\sigma_{ij} = -p\delta_{ij}$$

where p=pressure

$$p = -\mathrm{K}\Delta$$

where:

K=bulk modulus = $\sqrt{\lambda + \frac{2}{3}\mu}$ normally, but in the acoustic case $K = \sqrt{\lambda}$ because μ=0

$\Delta = \nabla.u =$ dilatation / volume change

For a plane wave $P = \exp\{i(\kappa.x - \omega t)\} = \exp\{i(k_x x + k_z z - \omega t)\}$ assuming no changes in the y direction.

$$\ddot{P} = c^2 \nabla^2 p$$

where *c*=wave speed

$$\ddot{P} = -\omega^2 p$$
$$\nabla^2 p = -(k_x^2 + k_z^2)p$$

The wave equation

$$-\left(\frac{\omega}{c}\right)^2 + (k_x^2 + k_z^2) = p = 0$$
$$(k_x^2 + k_z^2) = k^2 = \left(\frac{\omega}{c}\right)^2$$

Therefore pressure waves satisfy the wave equation and we also have the dispersion relationship as before.

The equation of motion

Navier's Equation:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial j \sigma_{ij}$$
$$= -\partial_j p \delta_{ij}$$
$$= -\partial_i p$$

$\rho \ddot{u} = -\nabla p$ **a pressure gradient**

Going back to the wave equation

$$\ddot{P} = \frac{\partial^2 p}{\partial t^2} = -k\ddot{\Delta} = -k(\nabla.\ddot{u})$$
$$\ddot{P} = k\nabla\left(\frac{1}{\rho}\nabla p\right)$$

For a homogeneous medium $\nabla\left(\frac{1}{p}\right) = 0$

So the wave equation becomes

$$\boxed{\ddot{P} = \frac{k}{\rho}\nabla^2 p}$$

where $\frac{k}{\rho} = c^2$

If we look at displacement instead, $U \cong \exp\{i(k.x - \omega t)\}$

We know $\rho\ddot{u} = -\nabla p$ from the Equation of Motion.

Then

$$\rho\ddot{u} = -\rho\omega^2 u = -\nabla p$$

$$\boxed{u = \frac{1}{\rho\omega^2}\nabla p}$$

$$u = (u_x, u_z) = \frac{1}{\rho\omega^2}\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial z}\right)$$