3D Topology and Data Structures - a Review and a Proposal

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1. A Review

1a. Spatial Models

There are at least two different views of space:
1. Space is empty and just littered with objects. Each object is represented individually.
2. Space is continuous. Used heavily in the geosciences to study the variability of a phenomenon (called a field), e.g. terrain elevation, water salinity or % gold in rock.

View 1 of space is what is used in vector-based GIS, and in 3D is often used for the modelling a city with separate, unattached buildings.

View 2 is what geology, for example, is about. We need to discretize a field to model it into a computer. A continuous field can be decomposed:
1. Regularly: elements having same shape
2. Irregularly: with simplices (e.g a Delaunay triangulation or tetrahedralization), or with arbitrary polyhedra.

1b. Spatial Data Structures

We must also differentiate between spatial data models and spatial data structures – a spatial model is implemented using a selected spatial data structure. We would classify the main 3D data structures as follows:

A: 2D (surface) decomposition (b-rep, for a single solid object)
1. half-edge of Mäntylä (1988)
2. winged-edge of Weiler (1988)
3. quad-edge of Guibas and Stolfi (1985)
4. Triangulation (TIN)
5. DCEL
6. GIS topological models (Zlatanova et al., 2004)

B: CSG (Constructive Solid Geometry – Boolean combinations of simple solids)

C: Regular 3D (volume) decomposition
1. voxel
2. octree (Samet, 1990)

D: Irregular 3D (volume) decomposition
1. facet-edge of Dobkin and Laszlo (1989) and its extension to any dimensions (Brisson, 1989)
2. G-maps of Lienhardt (1994) and Bertrand et al. (1993)

3. generalisation of half-edge to 3D (Lopes and Tavares, 1997)
4. simple tetrahedralization
5. Augmented Quad-Edge (this is our new proposal)

E: Non-manifold 3D structures (e.g. DeFloriani and Hui, 2003)

1c. Applications Domains

1. **Computer Graphics**: data structures such as octrees, k-d trees and BSP (binary-space partitioning) trees are used. These are not "topological" in the sense that object-face relationships are preserved.
2. **Computer Games:** CAD type systems are used to construct topologically-connected surface models of monsters, etc. At the display stage, speed requirements usually mean that the individual triangular elements are output to the computer graphics rendering engine.
3. **GIS Topological Models:** these are usually 2D. In 3D, most of the work done for modelling cities uses vector-based models. Each building or object is represented with a b-rep. Most of the efforts of this community is to develop models to store individual objects and to detect 'topological relationships' between 3D objects, i.e. to know where and if objects touch/intersect each other. Doing this assumes that somehow objects are in 2 different datasets and we want to overlay them, or that only individual objects are stored in a DB and topological relationships are restored 'on-the-fly'.
4. **Computational geometry:** specialized data structures are developed for many individual algorithmic problems. Many of the techniques developed have spilled over into other domains.
5. **CAD systems:** we are building complex models (buildings, aircraft, engines). There are two approaches: solid modelling (CSG) and boundary modelling. There are no built-in topological relationships and the simple primitive solids probably make this less interesting in GIS. Boundary modelling (b-rep) involves representing the solid object as a set of bounding faces. Almost of necessity this requires information about the connectivity of the faces. "Euler Operators" sit on top of the data structure used, to guarantee valid local surface modification.

Various more specific applications include: medical, geology, oceanography, architecture, molecular biology, GIS, kinetic models (require locally-updatable topological data structures), volumetric display, and city models.

2. A Proposal

We would like to generate a "complete" 3D topological data structure that could integrate at least Categories A and D of our brief review. In 2D, (Guibas and Stolfi, 1985) showed the importance and elegance of preserving both the primal and the dual of a graph on a 2-manifold within the same data structure. All of the 2D data structures in Category A may be represented by their quad-edge structure. Category B is different from the others in that there is no "topology" involved, and it will be ignored here. Category C, regular spatial subdivision, may be considered to be either an ordered set of unconnected blocks, or else a special case of Category D, which is a set of completely defined, space-filling polyhedra. Each polyhedral shell boundary may be defined by any Category A structure, modified to accommodate multiple use of adjacent faces, edges and vertices. A variety of (very complex) structures are given in the literature.

We have developed a simple extension of the quad-edge structure that links together the space-filling polyhedra by using their 3D dual graph. This therefore integrates Category A and D data structures (as well as C if required). We are still exploring if it is appropriate for non-manifold models (Category E), where dangling edges and faces may exist. The proposed approach preserves the lower dimensionality (2D) navigation and construction operations, as well as keeping the underlying quad-edge properties in 3D. These properties are: having a directly navigable structure, based on graph edges, that forms an edge algebra, and an atomic edge element that allows the preservation of attributes associated with 0-cells, 1-cells, 2-cells (and, in our case, 3-cells) - at the cost of somewhat higher storage by comparison with other techniques.

We achieve this by exploiting the relationships between the primal and dual structures, and constructing links between the two. This allows all attribute information to be associated exclusively with nodes and edges in the primal and dual space, permitting direct graph methods of analysis and navigation to be performed on any 3D space-filling structure. The structure is also locally modifiable, and has been tested for Voronoi diagrams and Delaunay triangulations. Earlier work (Merrett et al., 2002) suggests that quad-edges, being an algebra, may be stored effectively using relational databases, but this is still being examined. We believe that the structure is of general utility, is relatively simple to implement, and may form a common framework for further work.

References

BERTRAND, Y., J. DUFOURD, J. FRANÇON, and P. LIENHARDT, 1993. Algebraic specification and development in geometric modeling. In *Proceedings TAPSOFT'93*, volume 668 of LNCS, pages 75–89, Orsay, France.

BRISSON, E., 1989. Representing geometric structures in d dimensions: topology and order. In *Proceedings 5th Annual Symposium on Computational Geometry*, pages 218–227, Saarbruchen, West Germany. ACM Press.

DE FLORIANI, L. and A. HUI, 2003. A scalable data structure for three-dimensional non-manifold objects. In *Proceedings 1st Eurographics Symposium on Geometry Processing*, pages 72–82, Aachen, Germany.

DOBKIN, D. P. and M. J. LASZLO, 1989. Primitives for the manipulation of three-dimensional subdivisions. *Algorithmica*, 4:3–32.

GUIBAS, L. and J. STOLFI, 1985. Primitives for the manipulation of general subdivisions and the computation of Voronoi diagrams. *ACM Transactions on Graphics*, 4:74–123.

LIENHARDT, P., 1994. N-dimensional generalized combinatorial maps and cellular quasi-manifolds. *International Journal of Computational Geometry and Applications*, 4(3):275–324.

LOPES, H. and G. TAVARES, 1997. Structural operators for modeling 3-manifolds. In *Proceedings 4th ACM Symposium on Solid Modeling and Applications*, pages 10–18, Atlanta, Georgia, USA.

MÄNTYLÄ, M., 1988. *An introduction to solid modeling*. Computer Science Press, New York, USA.

MERRETT, T. M., Y. BÉDARD, D. COLEMAN, J. HAN, B. MOULIN, B. NICKERSON, and C. TAO, 2002. A tutorial on database technology for geospatial applications. Unpublished manuscript.

SAMET, H., 1990. *The Design and Analysis of Spatial Data Structures*. Addison-Wesley Publishing Company, Reading, Massachusetts, USA.

WEILER, K., 1988. The Radial Edge Structure: A topological representation for nonmanifold boundary modeling. In *Geometric Modeling for CAD Applications*. Elsevier Amsterdam.

ZLATANOVA, S., A. A. RAHMAN, and W. SHI, 2004. Topological models and frameworks objects. *Computers, Environment and Urban Systems,* 30:419–428.

Biography

Chris Gold held an Industrial Chair in GIS applied to forestry at Laval University, Quebec, from 1990-1999 and was Professor of GIS at the Hong Kong Polytechnic University from 2000-2004. He now holds an EU Marie Curie Chair in GIS at the University of Glamorgan. His primary interests are in topological structures and visualization.