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An Improved Single-Slice Rebinning Algorithm for Helical Cone-Beam CT

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Abstract: Large pitch was used to speed up scanning in helical cone-beam CT, but its reconstruction remains a challenging problem. The Single-Slice ReBinning method (SSRB), proposed by Noo et al, can obtain high image quality at large pitch, but artifacts appear when pitch is too large for given detector. In this paper, a new approximate reconstruction scheme, i.e. Improved Single-Slice ReBinning method (ISSRB) is described. It modified the key rebinning step of SSRB. Theoretical considerations and reconstruction of simulation data are presented in comparison to SSRB in the paper. Results show that the pitch of proposed method is ($1+\tan^2\delta$) times larger than that of SSRB for given detector size, and the reconstruction image quality is comparable or even better.

Key words: computed tomography; helical cone-beam; single-slice rebinning; image reconstruction

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In 1990s, helical CT was introduced to achieve higher volume coverage[1-2]. Then it combined with two-dimensional (2D) detector with Cone-Beam (CB) collimation of x-ray source, known as helical CB tomography. This approach increases the throughput of objects, and image quality is also improved[1].

But image reconstruction for cone-beam projections acquired along helical path is a particularly challenging problem. At first, 360° and 180° linear interpolation methods were used as starting points for spiral CT reconstruction and then extended to helical cone-beam CT[3-4]. The extended methods, however, had serious image quality problem due to discontinuous changeovers and existence of cone angle. In 1984, a 3D reconstruction algorithm was given by Feldkamp et al[5] for circular circle and then generalized to more complex path such as helix[6]. This was regarded as a mile stone in cone-beam reconstruction. It's approximate but suitable for larger cone angle and short object. Meanwhile, exact algorithms were proposed, but most of them are based on computing the Radon transform for a plane determined by the spiral path of the source[7-8]. Though accurate, exact algorithms are computationally intensive and require considerable amount of memory. Practically, approximate algorithms are easier to implement and computationally less demanding although producing artifacts. They are more efficient and preferable for technique utilization.

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The Single-Slice ReBinning algorithm (SSRB), put forward by Noo et al 1999[9], was a typical approximate helical cone-beam reconstruction method. It first rebins the cone-beam ray-sum to fan-beam projection data to a defined plane, and then is performed using 2D fan-beam Filtered BackProjection (FBP). The reconstruction image quality was said to be surprisingly good, even when the pitch of the helix is large[9]. Marc Kachelrieß et al and others proposed Advanced SSRB algorithm (ASSR) to cope with large cone-beam and pitch by tilted or nutated reconstruction plane[10-11], but it needs interpolation to reformat from reconstruction coordinates to cartesian coordinates after reconstructed.

In this paper, we present an improved single-slice rebinning algorithm. Results from simulation data show that the algorithm suits for larger pitch for given detector size and holds comparable or even better reconstruction quality than SSRB does.

1 Theory and algorithm development

1.1 Scanner geometry

To simplify the exposition, the z-axis will be referred to as the vertical direction. The source to z-axis distance is R, the pitch of the helix is P, and the distance between source and detector is D. The Field Of View (FOV) is an imaginary cylinder of radius r, centered on the z-axis. (see fig.1)

The locus of the source is described in a cylindrical coordinate system (R, $h(\beta)$, β). β is the projection angle. $h(\beta)$ denotes the vertical distance from the source to the reference horizontal plane where the scan started. When β increases by 2π, the source moves vertically upwards by distance P.

We employ $p_\beta(a, b)$ to represent the CB projection at projection angle β, and a, b refer to detector locations with the b-axis parallel to the z-axis and origin is the middle point of detector.

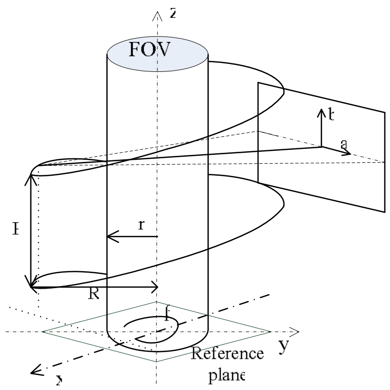

Fig.1 Scanner geometry

1.2 Rebinning of SSRB algorithm

As most 2D approaches for 3D reconstruction, SSRB converts the CB data set to a stack of fan-beam sinogram $p_{z_0}^F(\beta, a)$ by rebinning, where z_0 is vertical position the plane to be reconstructed and superscript F stands for fan-beam. Each sinogram is regarded as projections on one defined horizontal plane (z_0) of the FOV. Once the fan beam sinogram is obtained, reconstruction is performed using 2D FBP. For SSRB, the rebinning step only involves the vertical direction, each view of projection is estimated from a single oblique (CB) ray-sum passing through the nearest CB source location in the same projection angle and the middle point(M)of the intersection of the fan-line with the ROV[9]. As illustrated in fig.2(a)[9] and 2(b). Mathematically, the rebinning equation is:

$$p^F_{z_0}(\beta,a) = p_\beta(a,b)|_{b=b(z_0,a,\beta)} \tag{1}$$

Where
$$b(z_0,a,\beta) = \frac{a^2+D^2}{RD}\Delta z = \frac{a^2+D^2}{RD}(z_0 - h(\beta)) \tag{2}$$

And then $p^F_{z_0}(\beta,a)$ is filtered and backprojected:

$$f_{\text{SSRB}}(x,y,z_0) = \int_{\beta_{z_0}-\pi}^{\beta_{z_0}+\pi} \frac{R^2}{(R + x\cos\beta + y\sin\beta)^2} \frac{\sqrt{a^2+D^2}}{\sqrt{a^2+b^2+D^2}} p^F_{z_0}(\beta,a) * g(a)\ \mathrm{d}\beta \tag{3}$$

It can be concluded that, in SSRB, for given z-slice z_0 and projection angle β, the points (M) that the CB ray-sum passed are on an arc passing through the virtual fan-beam point(S) and center of FOV(O) with radius R. The $p^F_{z_0}(\beta,a)$ is rebinned from different coordinate b when a differs. As the fig.2(a, c) illustrated.

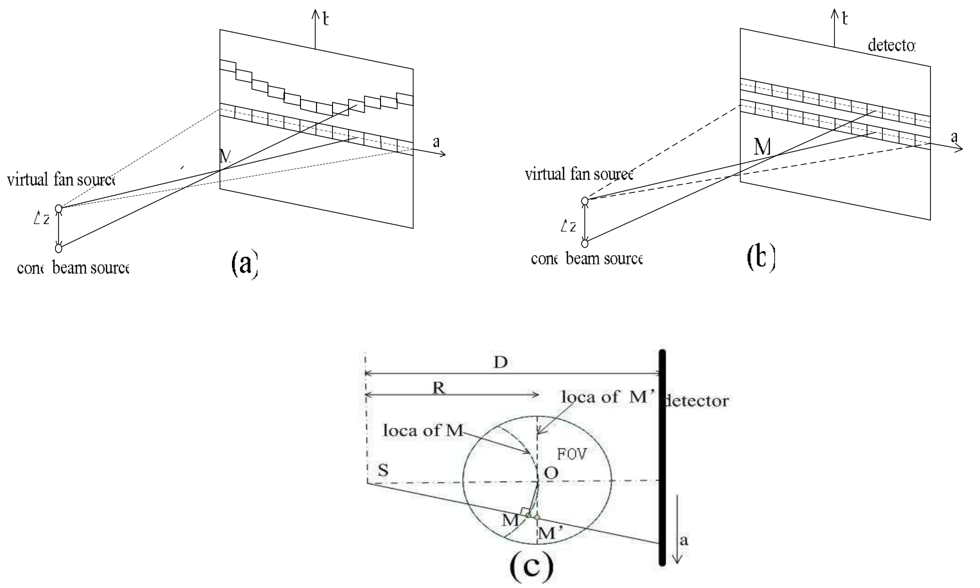

Fig.2　(a) Rebinning of SSRB, (b) Rebinning of ISSRB, Each fan-beam ray is estimated by the value of a CB ray emitted by a source directly above or below the virtual fan source. (c) The locus of the points that the selected CB ray passes through the reconstruction plane. In SSRB, the CB ray passes through the middle point M of the intersection of the fan-line with the ROI, but in ISSRB, the selected CB ray passes through the point (M') of the intersection of fan-line and line passing through the center of ROI and parallel to a-axis

1.3　Rebinning of ISSRB algorithm

The ISSRB rebinning step only involves vertical direction as well, but each fan-beam ray-sum is estimated from the oblique (CB) ray-sum passing through point M', the intersection of the fan-line and the line passing through center of ROI and parallel to *a*-axis. As the fig.2 (b) and (c) show:

Mathematically, the reconstruction is the same as equation (1) and (3), but

$$b(z_0, a, \beta) = \frac{D}{R}\Delta z = \frac{D}{R}(z_0 - h(\beta)) \tag{4}$$

For given projection angle β, when a differs, the row coordinate b maintains unchanged. This is the main difference with SSRB.

For a full scan reconstruction, the rebinning needs CB projections in 2π segment. If we set the *z*-slice in the center point of the segment, the maximum distance between CB source and virtual fan beam source is $\Delta z=0.5P$. So the short-scan technique of 2D fan-beam tomography with Parker's normalization scheme[12] is used in this paper. Then, the maximum distance between a virtual fan-source and its approximating CB source is given by $\Delta z=0.5P(\pi+2\delta)/2\pi$. This further

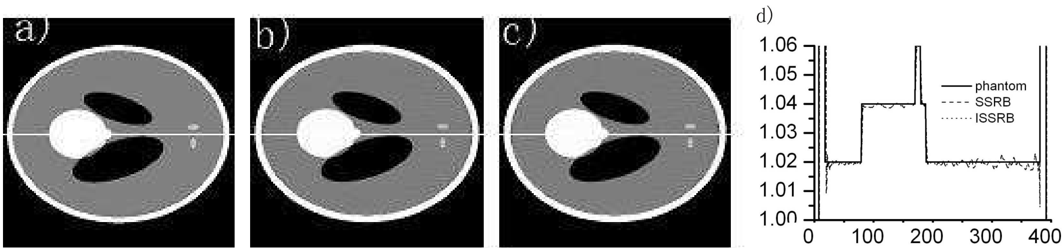

Fig.3 Slice z= −0.25 of the 3D Shepp phantom (a) reconstructed by SSRB (b) and ISSRB (c) with normalized pitch=5, left Grayscale[1.0-1.04]. (d) Central horizontal profiles of the reconstructions with SSRB and ISSRB

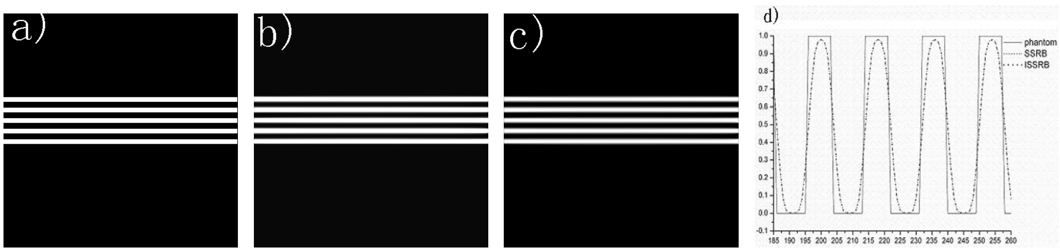

Fig.4 Slice x=0 of the 3D Disk phantom (a) reconstructed by SSRB (b) and ISSRB (c) with normalized pitch=5, Grayscale[0-1.0]. (d) Vertical profiles at left side of the reconstructions with SSRB and ISSRB

reduces the detector size required at large pitch.

2 Simulation and Results

In the simulation, the radius R of helical path was 600 mm. The detector was 1 000 mm away from the CB source. Each slice of detector had 321 cells in a direction with spacing of 3 mm and 10 rows in b direction with spacing of 5 mm. 180 projections were performed per 2π rotation.

Reconstructions were carried out on a grid of $400^2 \times 100$ voxels of size 1 mm. In projection, line integrals connecting sources point and the middle point of detector pixels were calculated analytically. The beam hardening, scattering and other effects are out of consideration. Reconstructions from simulation data are performed on two phantoms: the Shepp-Logan phantom(rotated for 90°) and the disk phantom (disk radius equals to 400mm). This is similar to the phantom used in [9].

In order to compare the image quality that can be achieved by SSRB and ISSRB, first, the simulation and reconstruction were performed with normalized pitch[14] equals to 5, that is, the distance moved is 15 mm per rotation. Second, the pitch P was set to be 45 mm. For the given 2D detector, some of CB ray-sum required for rebinning for SSRB are out of the detector, and this undoubtedly will degrade the reconstruction image quality, as will be discussed in the part 4. The results are shown in the fig.3, 4 and 5.

3　Discussion

This paper describes an improvement of the SSRB algorithm. The modification makes ISSRB approach more suitable for larger pitch for given detector and holds comparable, or even better reconstruction quality than SSRB does. This results in several improvements as follows.

3.1　Larger pitch for given detector size

In computed tomography, the coverage is of great importance, and this is largely determined by detector size and pitch in helical cone-beam CT. The pitch of SSRB for short-scan reconstruction is limited by the axial extent $2b_{max}$ of the area detector[9]. The relation is as bellow:

$$P \le \frac{2b_{\max}R}{D(1+\tan^2\delta)}\frac{2\pi}{\pi+2\delta} \tag{9}$$

Where $\delta = \arcsin(r/R)$ and 2δ is called the full fan-angle.

For ISSRB, the relation changes to:

$$P \le \frac{2b_{\max}R}{D}\frac{2\pi}{\pi+2\delta} \tag{10}$$

Comparing Eq.(9) and (10), we can conclude that, for a given detector size(axial extent $2b_{\max}$), the maximum P of ISSRB is about $(1+\tan^2\delta)$ times larger than that of SSRB. This increases the scan speed and coverage in axial direction ,or for a given pitch, the detector required is smaller. This is particularly significant for long and large object scanning, for example $\delta = \pi/4$.

3.2　Comparable or better image quality

Image quality between SSRB and ISSRB is not legible for small pitch. Both algorithms can adequately reconstruct image with high quality, as Fig.3 and 4 illustrated. This is because the Δz in equation (2) and (4) is small and the difference of b is not significant. But when pitch is large, ISSRB yields more uniform image than SSRB does, though both have artifacts. It can be saw(Fig.5) that there exists significant intensity drop at the edge of region by SSRB, which result from the lack of reconstruction data.

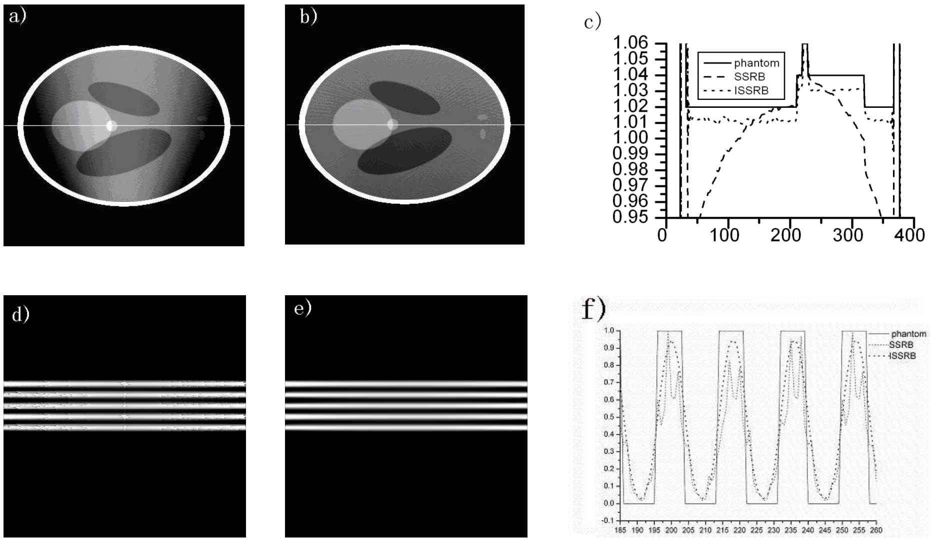

Fig.5 Slice z=0 of the 3D Shepp-Logan phantom reconstructed by SSRB (a) and ISSRB (b) with normalized pitch=15, Grayscale[0.96-1.06]. (c) Central horizontal profiles of the reconstructions with SSRB and ISSRB. Slice x=0 of the 3D Disk phantom reconstructed by SSRB (d) and ISSRB (e) with normalized pitch=15, Grayscale[0-1.0]. (f) Vertical profiles at left side of the reconstructions with SSRB and ISSRB

4 Conclusion

The reconstruction of cone-beam projections acquired with a helical vertex path is of primary importance for developments of computerized tomography. In this paper, we described and implemented an improvement of single-slice rebinning algorithm of helical cone-beam CT. Such an improvement involves in the step of rebinning. The major advantage of the proposed algorithm is that it satisfies larger pitch for given detector size and enlarges the coverage of helical cone-beam CT. Meanwhile, theoretical and numerical studies demonstrate that the improved algorithm maintains the comparable or better image quality than SSRB does.

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一种改进的锥束螺旋 CT 单层重排重建算法

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摘要：多层螺旋 CT 中，采用大螺距可以大大提高扫描速度，在工业和医学中颇受青睐。但在给定探测器尺寸条件下，螺距过大会出现明显的伪影。Noo 等 1999 年提出的单层重排算法（SSRB）能够适应较大的螺距，且有较高的重建图像质量。本文采用新的重排方法对 SSRB 算法进行了改进，理论分析及仿真实验表明，相同探测器大小时，改进的 SSRB 算法（ISSRB）可适用的螺距是 SSRB 的（$1+\tan^2\delta$）倍（δ 射线源半张角），且有相近或更高的图像质量。

关键词：锥束 CT；单层重排；螺距；图像质量

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