Structural Equation Modeling Example Using WinAMOS

The Wheaton Study

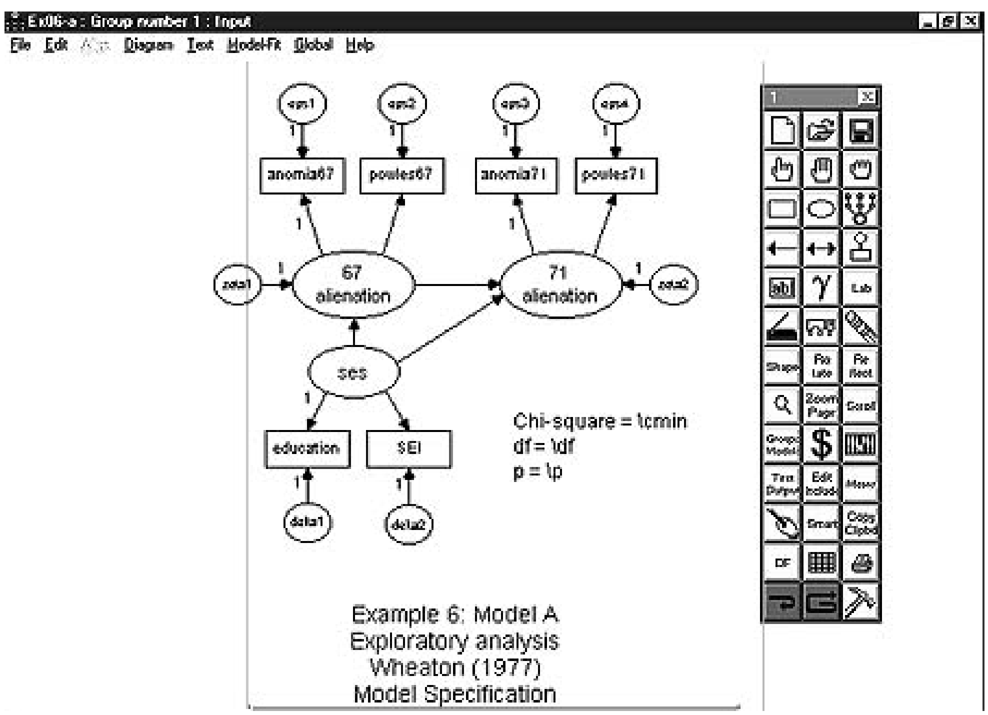
SEM is capable of a wide variety of output, as for assessing regression models, factor models, ANCOVA models, bootsrapping, and more. This particular output uses the Windows PC version of AMOS (WinAmos 3.51) for an example provided with the package, Wheaton's longitudinal study of social alienation. As such it treats regression with time-dependent data which may involve autocorrelation. (See B. Wheaton et al., "Assessing reliability and stability in panel models," pp. 84-136 in David R. Heise et al., *Sociological Methodology 1977*, San Francisco: Jossey-Bass, 1977).

The Wheaton study dealt with three latent variables, each measured by two indicators. Alienation67 was measured by anomia67 (a 1967 score on an anomia scale) and powles67 (a 1967 score on a powerlessness scale). Alienation71 was the same, but for two corresponding scales given in 1971. The third latent variable, SES (socio-economic status) was measured by education (years of schooling as of 1967) and SEI (Duncan's Socioeconomic Index, as of 1967).

WinAmos may be run in either text or graphics mode. The discussion below assumes that WinAmos has been launched in the graphics mode.

SEM Steps

1. **Loading in the data.** WinAMOS provides the Wheaton dataset in the file ex06-a.amw. Use File, Open, and select this file. In graphics mode the file will come up as below. Although pre-defined here, the graphics mode allows the researcher to create new models graphically, by adding circles for variables, arrows, and other elements you see below.



   In addition to the model, the Amos toolbar is show on the right-hand side of the window.

2. **Achieving identification of the model**. The variance of the latent variables and the regression (path) coefficients associated with them depend on the units with which the variables are measured, but initially this is unknown. For each latent variable and also for the unknown error terms, it is necessary to assign an arbitrary value to a regression weight associated with the latent variable or error term. Once this is done, the remaining coefficients can be estimated for the remaining paths in the model. Therefore, for each latent variable, one of the paths leading away from it toward one of its indicator measures has been set to 1 by the researcher. This sets the measurement scale of each latent variable, whereas without this the scale would be indeterminate. Likewise, the paths from each error term to each indicator variable are set at 1. With these constraints, the model is identified.

3. **Text Mode**. This graphics-mode specification of the model is equivalent to the following text-mode specification, which is the contents of the input file ex06-a.ami for the text mode:

```
Example 6, Model A:
Exploratory analysis

Stability of alienation, mediated by ses.
Correlations, standard deviations and
means from Wheaton et al. (1977).

$Mods=4

$Structure
     anomia67 <--- 67_alienation (1)
     anomia67 <--- eps1 (1)
     powles67 <--- 67_alienation
     powles67 <--- eps2 (1)

     anomia71 <--- 71_alienation (1)
     anomia71 <--- eps3 (1)
     powles71 <--- 71_alienation
     powles71 <--- eps4 (1)

     67_alienation <--- ses
     67_alienation <--- zeta1 (1)

     71_alienation <--- 67_alienation
     71_alienation <--- ses
     71_alienation <--- zeta2 (1)

     education <--- ses (1)
     education <--- delta1 (1)
     SEI <--- ses
     SEI <--- delta2 (1)

$Include = wheaton.amd
```

4. **Testing the model**. Once the model has been specified properly, the analysis can be run in graphics mode by clicking on the Calculate Estimates (abacus) icon in the toolbar. The output looks like this. Instructor comments are in blue and are not part of the WinAMOS output.

```
Example 6, Model A:                                          Page 1

User-selected options
```

```
--------------------

Output:

         Maximum Likelihood
```

Note SEM uses maximum likelihood, not ordinary least squares in estimating the model. OLS seeks to minimize the sum of squared distances of the data points to the regression line. MLE seeks to maximize the log likelihood, LL, which reflects how likely it is (the odds) that the observed values of the dependent may be predicted from the observed values of the independents.

```
Output format options:

         Compressed output

Minimization options:

         Technical output
         Modification indices at or above 4.000000
         Machine-readable output file

Sample size:   932


Your model contains the following variables


             anomia67                       observed   endogenous
             powles67                       observed   endogenous
             anomia71                       observed   endogenous
             powles71                       observed   endogenous
             education                      observed   endogenous
             SEI                            observed   endogenous

             71_alienation                  unobserved endogenous
             67_alienation                  unobserved endogenous

             eps1                           unobserved exogenous
             eps2                           unobserved exogenous
             eps3                           unobserved exogenous
             eps4                           unobserved exogenous
             ses                            unobserved exogenous
             delta1                         unobserved exogenous
             zeta1                          unobserved exogenous
             zeta2                          unobserved exogenous
             delta2                         unobserved exogenous


                     Number of variables in your model:   17
                     Number of observed variables:         6
                     Number of unobserved variables:      11
                     Number of exogenous variables:        9
                     Number of endogenous variables:       8
```

```
Summary of Parameters
```

The 11 fixed weights below are the 1's specified in step 2 above.

	Weights	Covariances	Variances	Means	Intercepts	Total
Fixed:	11	0	0	0	0	11
Labeled:	0	0	0	0	0	0
Unlabeled:	6	0	9	0	0	15
Total:	17	0	9	0	0	26

```
The model is recursive.

Model: Your_model

Computation of Degrees of Freedom
```

The 21 "sample moments" are the 6 sample variances of the 6 indicators, and their 15 covariances. The 15 parameters are the 6 regression weights and 9 variances to be estimated for the model. The 6 degrees of freedom is the difference between these two numbers.

```
                      Number of distinct sample moments:   21
          Number of distinct parameters to be estimated:   15
                                           ------------------------
                                        Degrees of freedom:    6
```

Maximum likelihood estimation is an iterative process. The table below gives a history of the iterations. This is a technical option in the output and is unlikely to be used directly by the researcher.

```
Minimization History
    0e  5  0.0e+00  -2.2608e-01   1.00e+04    2.51961429836e+03    0  1.00e+04
    1e  2  0.0e+00  -3.4582e-02   1.69e+00    7.68466803623e+02   20  7.84e-01
    2e  1  0.0e+00  -2.8236e-02   6.93e-01    2.04172787908e+02    5  7.78e-01
    3e  0  2.3e+02   0.0000e+00   5.10e-01    9.26387843078e+01    6  7.67e-01
    4e  0  2.7e+01   0.0000e+00   4.98e-01    8.37944324493e+01    2  0.00e+00
    5e  0  3.0e+01   0.0000e+00   2.60e-01    7.23390148175e+01    1  1.06e+00
    6e  0  3.3e+01   0.0000e+00   5.56e-02    7.15511793962e+01    1  1.04e+00
    7e  0  3.4e+01   0.0000e+00   7.39e-03    7.15437747647e+01    1  1.01e+00
    8e  0  3.4e+01   0.0000e+00   7.73e-05    7.15437737196e+01    1  1.00e+00
Minimum was achieved
```

Chi-square fit index: This is the most common fit test, printed by all computer programs. AMOS and LISREL refer to this simply as *chi-square*, and others call it both *chi-square goodness of fit* and *chi-square badness-of-fit*. The chi-square fit index tests the hypothesis that an unconstrained model fits the covariance/correlation matrix as well as the given model. The chi-square value should not be significant if there is a good model fit. In this case, the model is rejected as not being a good fit with the data. A problem with this test is that the larger the sample size, the more likely the rejection of the model and the more likely a Type II error. The chi-square fit index is also very sensitive to violations of the assumption of multivariate normality.

```
Chi-square =    71.544
Degrees of freedom =    6
Probability level =     0.000
```

The MLE estimates of the regression weights below are the estimated path coefficients for the arrows in the model. In order to identify the model, some of these were fixed beforehand as 1.000 in Step 2 (ex., the path from the latent variable 67_alienation to the indicator variable anomia67). Standard errors are also given for the path coefficients. "C.R." is the *critical ratio*, which is the estimate divided by its standard error. If we are dealing with random sample variables with standard normal distributions, estimates with critical ratios more than 1.96 are significant at the .05 level. Estimates, standard errors, and critical ratios are given further below for the variances of variables in the model.

```
Maximum Likelihood Estimates
----------------------------

Regression Weights:                    Estimate     S.E.      C.R.     Label
-------------------                    --------   -------   -------   -------

67_alienation <-------------- ses       -0.614     0.056   -10.876
71_alienation <---- 67_alienation        0.705     0.054    13.163
71_alienation <-------------- ses       -0.174     0.054    -3.234
powles71 <--------- 71_alienation        0.849     0.040    21.243
anomia71 <--------- 71_alienation        1.000
powles67 <--------- 67_alienation        0.888     0.041    21.413
anomia67 <--------- 67_alienation        1.000
education <------------------ ses        1.000
SEI <------------------------ ses        5.331     0.430    12.403

Variances:                             Estimate     S.E.      C.R.     Label
----------                             --------   -------   -------   -------
                                  ses    6.663     0.641    10.398
                                zeta1    5.307     0.473    11.230
                                zeta2    3.741     0.388     9.653
                                 eps1    4.014     0.343    11.700
                                 eps2    3.191     0.271    11.757
                                 eps3    3.700     0.373     9.908
                                 eps4    3.625     0.292    12.414
                               delta1    2.947     0.500     5.900
                               delta2  260.910    18.241    14.304
```

Modification indexes (MI). The improvement in fit is measured by a reduction in chi-square, which makes the chi-square fit index less likely to be found significant (recall a finding of significance corresponds to rejecting the model as one which fits the data). For each fixed and constrained parameter (coefficient), the modification index reflects the predicted decrease in chi-square if a single fixed parameter or equality constraint is removed from the model by eliminating its path, and the model is reestimated. The "Par Change" column, which stands for parameter change, gives the actual estimate of how much the coefficient would change.

In the case of modification indexes for covariances, the MI has to do with the decrease in chi-square if the two error term variables are allowed to correlate. In the case of MI for estimated regression weights, the MI has to do with the decrease in chi-square if the path between the two variables is eliminated, no longer requiring estimation of that weight in the model. One arbitrary rule of thumb is to consider eliminating paths associated with parameters whose modification index exceeds 100. However, another common path is simply to eliminate the parameter with the largest MI, then see the effect as measured by the chi-square fit index. Naturally, eliminating paths or allowing correlated error terms should only be done when it makes substantive as well as statistical sense to do so. LISREL and AMOS both compute modification indexes.

In this case, the largest MI is the 40.911 for the eps1 (error for anomia67) and eps3 (error for anomia71) error terms. This suggests dropping the constraint that the correlation of these two terms be zero. That is, allowing correlation will decrease chi-square by an estimated 40.911 points. The Wheaton data are panel data and in any time series, autocorrelation of the same measure (anomia) at two different time points (1967 and 1971) seems likely, so there is a sound theoretical reason for eliminating this constraint. The same logic applies to eliminating the zero-correlation constraint between eps2 and eps4 (the indicators for powerlessness in 1967 and 1971 respectively), which is estimated to reduce chi-square by 26.545. In this output, however, we have not rerun the model as respecified in this manner.

```
Modification Indices
--------------------

Covariances:                                      M.I.    Par Change
                                               ---------  ----------

       eps2 <--------------------> delta1        5.905      -0.424
       eps2 <----------------------> eps4       26.545       0.825
       eps2 <----------------------> eps3       32.071      -0.989
       eps1 <--------------------> delta1        4.609       0.421
       eps1 <----------------------> eps4       35.367      -1.070
       eps1 <----------------------> eps3       40.911       1.254


Variances:                                        M.I.    Par Change
                                               ---------  ----------

Regression Weights:                               M.I.    Par Change
                                               ---------  ----------

       powles71 <--------------- powles67        5.457       0.057
       powles71 <--------------- anomia67        9.006      -0.065
       anomia71 <--------------- powles67        6.775      -0.069
       anomia71 <--------------- anomia67       10.352       0.076
       powles67 <--------------- powles71        5.612       0.054
       powles67 <--------------- anomia71        7.278      -0.054
       anomia67 <--------------- powles71        7.706      -0.070
       anomia67 <--------------- anomia71        9.065       0.068
```

Measures of fit. Below, AMOS next prints out a large number of alternative measures of model fit. Each measure is calculated for three models. "Your model" is the model as specified by the researcher. The "independence model" is the model in which variables are assumed to be uncorrelated with the dependent(s), so if the fit for "your model" is no better than for the "independence model," then your model should certainly be rejected. The "saturated model" is one with no constraints and will always fit any data perfectly, so normally your model will have a measure of fit between the saturated and independence models.

NPAR is the number of parameters being estimated in the model and is not a measure of fit.
P(CMIN) deals with *minimum sample discrepancy*. If P(CMIN) is less than .05, we reject null hypothesis that the data are a perfect fit to the model. In practice, the null hypothesis is beside the point of most research and this measure is little used. For any sizable sample, the null hypothesis will likely be rejected. By this criterion the present model is rejected as being a perfect fit.
CMIN/DF is the minimum sample discrepancy divided by degrees of freedom. This is called *relative chi-square* or normal chi-square. Some researchers allow values as large as 5 as being an adequate fit, but conservative use calls for rejecting models with relative chi-square greater than 2 or 3. By this criterion the present model is rejected.

```
Summary of models
-----------------

               Model  NPAR        CMIN     DF           P     CMIN/DF
    ----------------  ----   ---------     --   ---------   ---------
          Your_model    15      71.544      6       0.000      11.924
     Saturated model    21       0.000      0
  Independence model     6    2131.790     15       0.000     142.119
```

RMR is the root mean square residual. RMR is the square root of the mean squared amount by which the <u>sample</u> variances and covariances differ from the corresponding <u>estimated</u> variances and covariances, estimated on the assumption that your model is correct. The smaller the RMR, the better the fit.
GFI is the Goodness of Fit Index. GFI varies from 0 to 1, but theoretically can yield meaningless negative values. By convention, GFI should by equal to or greater than .90 to accept the model. By this criterion the present model is accepted.
AGFI is the Adjusted Goodness of Fit Index. AGFI is a variant of GFI which uses mean squares instead of total sums of squares in the numerator and denominator of 1 - GFI. It, too, varies from 0 to 1, but theoretically can yield meaningless negative values.AGFI should also be at least .90. By this criterion the present model is accepted.
PGFI is the Parsimony Goodness of Fit Index. It is a variant of GFI which penalizes GFI by multiplying it times the ratio formed by the degrees of freedom in your model and degrees of freedom in the independence model.

```
               Model         RMR         GFI        AGFI        PGFI
    ----------------  ----------  ----------  ----------  ----------
          Your_model       0.284       0.975       0.913       0.279
     Saturated model       0.000       1.000
  Independence model      12.356       0.494       0.292       0.353
```

The next set of goodness of fit measures, below, compare your model to the fit of the independence model. Since the fit of the independence model is usually terrible, comparing your model to it will generally make your model look good but may not serve your research purposes. The DELTA and RHO headings are alternative names for these measures.
NFI is the normed fit index, which varies from 0 to 1, with 1 = perfect fit. By convention, NFI values below .90 indicate a need to respecify the model.
RFI is the relative fit index, which is <u>not</u> guaranteed to vary from 0 to 1. RFI close to 1 indicates a good fit.
IFI is the incremental fit index, which is <u>not</u> guaranteed to vary from 0 to 1.IFI close to 1 indicates a good fit and values

above .90 an acceptable fit.
TLI is the Tucker-Lewis coefficient,also called the Bentler-Bonett non-normed fit index (NNFI). TLI is not guaranteed to vary from 0 to 1. TLI close to 1 indicates a good fit.
CFI is the comparative fit index, which varies from 0 to 1. CFI close to 1 indicates a very good fit, and values above .90 an acceptable fit.

Model	DELTA1 NFI	RHO1 RFI	DELTA2 IFI	RHO2 TLI	CFI
Your_model	0.966	0.916	0.969	0.923	0.969
Saturated model	1.000		1.000		1.000
Independence model	0.000	0.000	0.000	0.000	0.000

PRATIO is the parsimony ratio, which is the ratio of the degrees of freedom in your model to degrees of freedom in the independence (null) model. PRATIO is not a goodness-of-fit test itself, but is used in goodness-of-fit measures like PNFI and PCFI which reward parsimonious models (models with relatively few parameters to estimate in relation to the number of variables and relationships in the model.
PNFI is the parsimony normed fit index, equal to the PRATIO times NFI.
PCFI is the parsimony comparative fit index, equal to PRATIO times CFI.

Model	PRATIO	PNFI	PCFI
Your_model	0.400	0.387	0.388
Saturated model	0.000	0.000	0.000
Independence model	1.000	0.000	0.000

NCP is the noncentrality parameter. It and FO are used in the computation of RMSEA, the root mean square error of approximation, which incorporates the discrepancy function criterion (comparing observed and predicted covariance matrices) and the parsimony criterion (see above). For each, LO 90 and HI 90 indicate 90% confidence limits on the coefficient. By convention, there is good model fit if RMSEA less than or equal to .05. There is adequate fit if RMSEA is less than or equal to .08. By this criterion, the model is rejected since RMSEA is .108. PCLOSE tests the null hypothesis that RMSEA is no greater than .05. Since PCLOSE is approximately zero, we reject the null hypothesis and conclude that RMSEA is greater than .05, indicating lack of a close fit.

Model	NCP	LO 90	HI 90
Your_model	65.544	41.936	96.603
Saturated model	0.000	0.000	0.000
Independence model	2116.790	1968.786	2272.133

Model	FMIN	F0	LO 90	HI 90
Your_model	0.077	0.070	0.045	0.104
Saturated model	0.000	0.000	0.000	0.000
Independence model	2.290	2.274	2.115	2.441

Model	RMSEA	LO 90	HI 90	PCLOSE
Your_model	0.108	0.087	0.132	0.000
Independence model	0.389	0.375	0.403	0.000

Next come a set of measures based on information theory. They are appropriate when comparing models which have been estimated using maximum likelihood estimation. As a group, this set of measures is less common in the literature.
AIC is the Akaike information criterion.
BCC is the Browne-Cudeck criterion.
BIC is the Bayes information criterion, also known as Akaike's Bayesian information criterion (ABIC).
CAIC is the consistent AIC criterion.
ECVI is another variant on AIC.
MECVI is a variant on BCC.

Model	AIC	BCC	BIC	CAIC
Your_model	101.544	101.771	200.980	189.104
Saturated model	42.000	42.318	181.211	164.584
Independence model	2143.790	2143.881	2183.565	2178.814

Model	ECVI	LO 90	HI 90	MECVI
Your_model	0.109	0.084	0.142	0.109
Saturated model	0.045	0.045	0.045	0.045
Independence model	2.303	2.144	2.470	2.303

Below is Hoelter's critical N. This is the largest sample size at which the researcher would accept the model at the .05 or .01 levels. This throws light on the chi-square fit index, which has the problem that the larger the sample size, the more likely the rejection of the model and the more likely a Type II error. In this case, actual sample size was 932 and the model was rejected. If the sample size had been only 164, it would have been accepted at the .05 level.

Model	HOELTER .05	HOELTER .01
Your_model	164	219
Independence model	11	14

```
Execution time summary:
          Minimization:     0.170
         Miscellaneous:     0.110
             Bootstrap:     0.000
                 Total:     0.280
```